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THE MODIFIED TRANSPARENT PLANT BALANCE METHOD OF LIFE ANALYSIS FOR INDUSTRIAL PROPERTY

by

© PRASANNA KARPUR

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled THE MODIFIED TRANSPARENT PLANT BALANCE METHOD OF LIFE ANALYSIS FOR INDUSTRIAL PROPERTY submitted by PRASANNA KARPUR in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE in MECHANICAL ENGINEERING.

DEDICATION

Dedicated to my parents,

Mr. Karpur Rama Rao

and

Mrs. Sudha Karpur.

ABSTRACT

In an industrial ambient, both regulatory and nonregulatory, an understanding of the mortality behavior of physical property is necessary to compute depreciation expense for revenue requirements and/or rate regulation purposes. The estimation of the mortality characteristics is commonly called life estimation. Life analysis is an important element of life estimation. The end result of life analysis is an estimate of the probable average service life, the probable retirement dispersion pattern and a knowledge of any discernible trends in the above two.

There are various methods of life analysis namely actuarial methods, semiactuarial methods and the simulation methods. The method of life analysis used is dependent on the type of available data.

Transparent Plant Balance Method (TPBM) is a simulation method of life analysis applicable when a partial and unaged data is available. The method was developed by Edmonton Telephones. A sensitivity analysis of the model conducted by A.Tharumarajah of the University of Alberta revealed several limitations of the model. The present study has been conducted to modify the Transparent Plant Balance Method so as to overcome some of the prevailing limitations of the model. This modified version has been called the MTPBM. A sensitivity analysis has been conducted to evaluate the performance of the MTPBM for varying input parameters.

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1. INTRODUCTION

1.1 Purpose

The purposes of this study are

1. to have a brief overview of the existing methods of life analysis of industrial property,
2. to evaluate and analyze the performance of the existing Transparent Plant Balance Method (developed by Edmonton Telephones) using the results of the study conducted by Tharumarajah [11],
3. to develop a modified version of the Transparent Plant Balance Method so as to overcome some of the prevailing limitations of the method (this modified version will be called the MTPBM), and
4. to study the performance of the newly developed MTPBM using stochastic data sets generated by a Monte Carlo Simulator developed for the purpose.

1.2 Background Information

This section gives a brief account of the existing methods of life analysis and its need.

1.2.1 Depreciation

Any property in use loses value over its life. Hence, a property which has been used is not worth the same as an identical new property. This phenomenon of the loss in value is called Depreciation. This phenomenon can be attributed to

three main factors:

1. physical factors,
2. functional factors, and
3. factors unrelated to the property.

Physical Factors

The physical factors for the loss in value and finally retirement can be due to:

1. the physical damage because of accidents like collisions, falls, breaking of machinery by extraneous forces etc.,
2. catastrophes like floods, storms, earthquake, fire etc.,
3. wear and tear from normal usage, and
4. deterioration over time inspite of maintenance and repairs. This might be due to the factors like decay of timber, action of chemicals, weathering, rusting, etc.

Functional Factors

Any industrial property is deemed functionally inefficient whenever the same function can be performed more efficiently and effectively by other equipment of similar or different kind. The functional inefficiency might be because of (i)inadequacy of the capacity of the equipment caused by an increased demand or reduced efficiency of the property, (ii)obsolescence resulting from the invention and/or

development of more effective performing models. Obsolescence may result from either economic factors or due to outdated style and mode.

Factors Unrelated to the Property

Occassionally the management may be forced to retire some property inspite of satisfactory performance. This may be due to: (i)the termination of the need for that kind of property, (ii)closing down or abandonment of the enterprise, and/or (iii)the requirement of a public authority like the municipality, provincial government or the Federal Government.

The revenues and profits of any organization can be determined only with a consideration of the depreciation expenses. A majority of the methods of calculating depreciation are dependent on the service life of the property being depreciated. The service life can be measured in units like years, units of production etc. A knowledge of the probable service life of the physical property is especially important in a regulatory ambient for the rate/price regulation and for the revenue requirements determination by the governmental authorities. The average service life and the retirement dispersion about the average service life are very useful in the managerial decision making processes like 'How much to buy?', 'Which kind of equipment to buy?', 'When to buy?' etc.

1.2.2 Life Analysis

There are three important steps of life estimation. They are,

1. data selection for the analysis,
2. life analysis based on the selected data,
   a. treatment of the data (development of the survivor curves),
   b. mathematical and/or graphical description of the life characterestics,
3. life forecast in light of the results obtained from the life analysis.

Life analysis is an important step of life estimation. A life analysis yields (i)the probable average service life (ASL), (ii)the probable dispersion pattern of the retirements about the average service life and (iii)discernible trends, if any, in the above two factors.

A life forecast seeks to predict future service lives based on informed judgement and past experience. In fact Edison Electric Institute has cautioned that the plant installed today might bear little or no resemblence to the plant being retired or which has already been retired. The same view is expressed by Winfrey [15] as under:

> 'While the author strongly recommends the development and use of the retirement data and the survivor curves as the basis of estimating the probable life of the property units, he does not mean that the expert judgement should be done away with in favor of pure statistical treatment. Each individual item, each group of items, and each property or company must be dealt with in the light of its present condition, its character and the

> amount of service production, and its relation to the present and the probable future economic trends, art of manufacture, and management policies. Tables of probable service lives, type survivor curves and statistical methods are simply means of recording the past experience to use in predicting what the future service might be.'

There are various methods of life analysis which can be broadly classified into three categories. They are,

1. Actuarial methods,
2. Semiactuarial methods, and
3. Simulation methods.

The choice of the method of life analysis is dictated by the type of available data. For the actuarial methods to be applicable, a complete aged data record showing the age of each unit at the retirement date and the age composition of the survivor group is very essential. If the data records are unaged and either complete or incomplete, one of the semiactuarial or simulation methods is used. While the semiactuarial analysis yields an estimation of only the average service life, the simulation methods of analysis provide estimates of both the probable average service life and the probable dispersion pattern of the individual retirements about the probable average service life.

The best method of service life estimation is to base the estimation on the study of the past experience. Life insurance companies have developed mortality tables by which the average life of humans and their expectancy of life at any age can be determined with accuracy. In a similar manner, the industrial statisticians and engineers have

studied the life histories and ages at retirement of many different types of the industrial physical property. This data enables one to forecast the probable lives of similar units that are still in service. It is likely that this estimation of life expectancy of any single unit or group of units will be in error. Nevertheless, if the service conditions of the property are considered by the engineers in addition to their own expert judgement, the probability of an error in the forecast of the life expectancy will be substantially reduced, especially if the estimate is revised from time to time as and when new data becomes available.

Accounting regulations, books on engineering valuation, and the utility regulations usually contain tables of estimated average service lives which can be used to establish a tentative figure for the average service life and the other mortality characteristics of the property for which the retirement records and experience are not available. There are many types of such tables and curves available today from different sources. Iowa Type Curves are one such type curves. These curves will be discussed in detail in one of the following chapters. These 'Iowa Type Curves' are the basis for almost all of the simulation methods.

The important simulation methods of life analysis are:

1. Simulated Plant Record Method (SPR),
2. Computed Mortality Method, and
3. Transparent Plant Balance Method (TPBM).

The first method mentioned above is used when only unaged but complete data is available while the last two methods are used when only unaged and incomplete data is available. Whereas the Simulated Plant Record method of life analysis is extensively researched and is in use, the Computed Mortality method and the Transparent Plant Balance Method are still in the research and developmental stages.

The Transparent Plant Balance Method was developed by Edmonton Telephones. A sensitivity analysis of the model was conducted by A.Tharumarajah [10,11]. Due to this sensitivity analysis, several limitations of the model have come to light. The present study is intended to overcome some of these limitations by developing a modified version of the model.

1.3 Scope and Methodology

This section provides a synopsis of the discussion to follow in the subsequent chapters.

1.3.1 Scope

The scope of this study includes:

1. a brief overview of the existing methods of life analysis,
2. a detailed discussion of the Transparent Plant Balance Method of life analysis,
3. development of a modified version of the Transparent Plant Balance Method so as to overcome some of the

existing limitations, and

4. a performance study of the newly developed version of the Transparent Plant Balance Method using stochastic data sets of known mortality charateristics generated from a Monte Carlo Simulator developed for this purpose.

1.3.2 Methodology

The methodology adopted for the study has been summarized in this section.

Chapter 2: Methods of Life Analysis

This chapter covers a brief overview of the various existing methods of life analysis like the actuarial methods, semiactuarial methods and the simulation methods (except the TPBM). For each method, a short outline of the process, its applicability and limitations will be discussed.

Chapter 3: TPBM

This chapter exclusively deals with the Transparent Plant Balance Method as developed by Edmonton Telephones and also the study conducted by Tharumrajah [11] to evaluate its performance.

Chapter 4: Modified TPBM

To begin with, this chapter evaluates the results of the study conducted by Tharumarajah [11]. This evaluation

helps to provide possible explanations for the occasional unpredictable behavior of the Transparent Plant Balance Method. With this as the basis, the modified version of the Transparent Plant Balance Method is developed.

Chapter 5: Monte Carlo Simulator

In this chapter, the discussion concentrates on the Monte Carlo Simulator developed to generate the data sets of known mortality characteristics, growth profile and growth rate. These data sets will be used in the performance tests to be conducted for the performance evaluation of the modfied version of the TPBM.

Chapter 6: Performance Evaluation

This chapter deals with the performance evaluation tests conducted on the model. A discussion on the sensitivity of the model to the changes in the Observation Band length, Transparent Band length and the growth profile will be presented in addition to a discussion of the behavior of the various indices.

Chapter 7: Summary and Conclusions

This chapter summarizes the present study, lists all the conclusions and finally discusses the scope for future research in the field.

2. METHODS OF LIFE ANALYSIS

As mentioned earlier in Chapter 1, there are many methods of life analysis. The choice of the method is influenced to a great extent by the kind of data available. In this chapter, many of these methods of analysis will be reviewed.

The existing methods of life analysis can be classified into 3 types, namely:

1. actuarial methods,
2. semiactuarial methods, and
3. simulation methods.

Before proceeding any further, a few definitions used in life analysis will be presented in the following section.

Terminology Used in Life Analysis

The following are a few definitions [14] used in conjunction with a survivor curve. These definitions have also been illustrated on Figure 2.1:

1. A Property Group (or Property Account) is a collection of similar units (usually for accounting purposes) comprising a property or a section of the property regardless of the ages of the individual units.
2. A Vintage Group (or Original Group) is a collection of similar units installed in service at the same time or in the same accounting period.
3. Original Data refers to the records showing showing kind of property installed, the number of units installed,

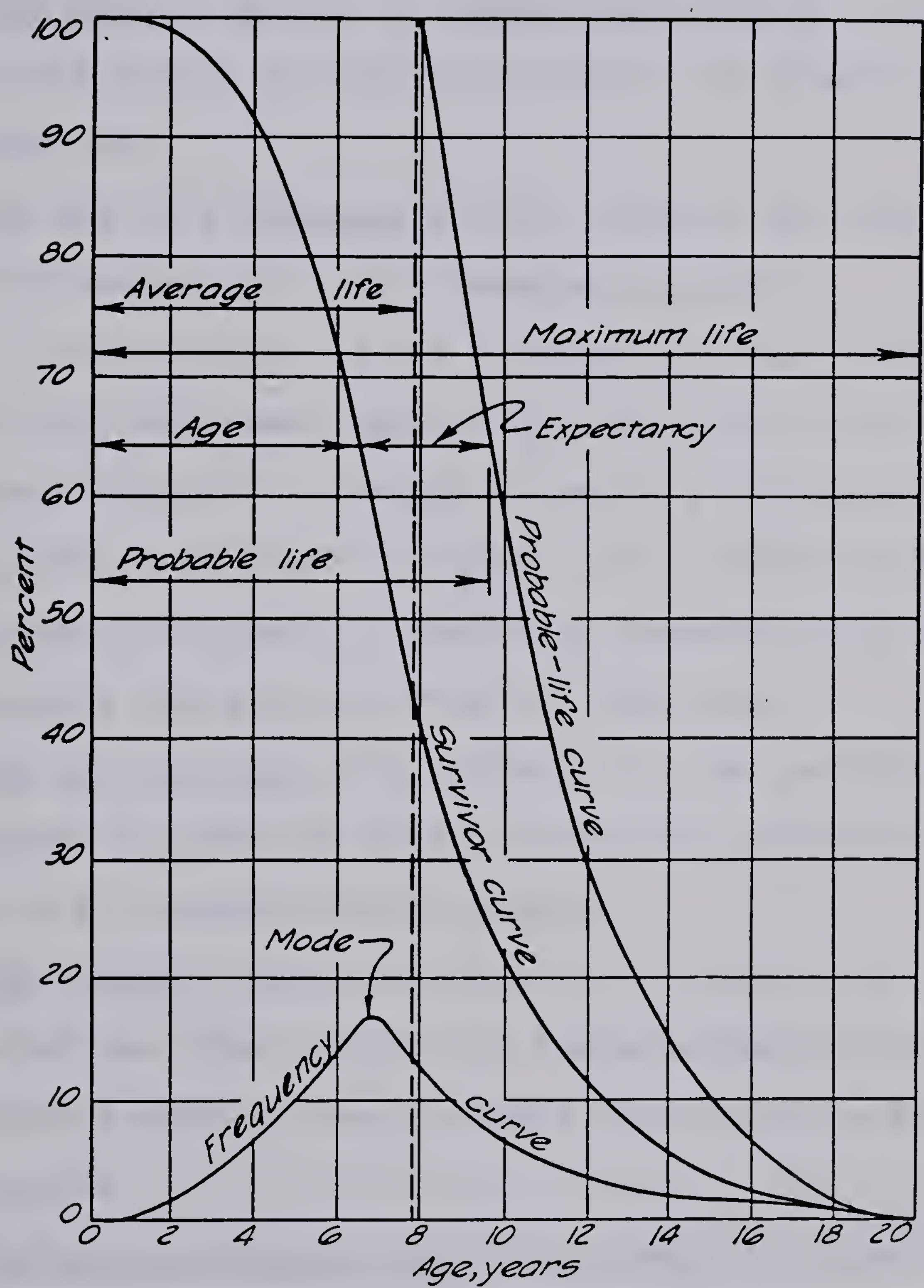


Figure 2.1 Typical Survivor Curve and the Associated Terminology [11]

dollar cost, ages, dates of placement in service, dates of retirement and other related factors necessary to a complete understanding of the life history of the property during the period covered by the data.

4. Observation period (or Experience Period) is comprised of the number of years over which the property group is observed.

5. The age of a property is the lapsed time from the date of installation to the observation date.

   The average age of a property group is the average of the individual ages of all the units in the group. For convenience, the age is usually designated to the nearest whole year on January 1st, which would age the property on the half year when measured from the assumed average installation date of July 1st.

6. The Service Life of a property is that period of time extending from the date of its installation to the date of its retirement from service.

7. The Probable Life of a unit is that period of time extending from its date of installation to the forecasted date when probably it will be retired from service.

8. The Average Service Life of a group of property is the quotient obtained by dividing the sum of the service lives of all the units by the number of units in the group. It is also equal to the area (in percent-years or unit-years) under the survivor curve divided by 100 (or

the total number of units).

9. <u>The Expectancy of Life</u> of a unit is that period of time extending from the observation age (usually the present) to the forecasted date when the unit probably will be retired from service. Age plus expectancy always equals the probable life.

10. <u>The Probable Average Service Life</u> of a group of units is the average of the probable service lives of the individual units in the group.

11. <u>The Maximum Life or Age</u> is the age of the last unit of a given group to be retired from service. It is also the age at which the survivor curve has a zero ordinate or zero percent surviving.

12. <u>Survivor Curves</u> show the property surviving in service at successive ages. At any particular age, the ordinate of the curve gives the percentage surviving (or the actual number of units) at that age. The abscissa is usually measured in either years or the age expressed as a percent of the average service life.

    The original survivor curve is the curve drawn through the points calculated from the original data without adjustment. Since the original survivor curve is generally irregular, it is smoothed to produce a smooth survivor curve, sometimes referred to as an adjusted curve.

13. <u>A Stub Survivor Curve</u> is an incomplete survivor curve; that is, one which does not extend to zero percent

surviving because of the lack of the retirement data.

14. A Probable Life Curve shows the probable average life of the survivors at any age from zero to the maximum life.
15. Retirement Frequency Curve shows the percent (or the number of units) retired during various age intervals.
16. Mode is the point on the frequency curve having the greatest ordinate.
17. Generalized Curves are those curves whose ordinates are expressed in percentage of the total number of units and whose abscissa (ages) are expressed in percentage of the average service life.
18. Type Curves depict typical survivor curves and frequency curves. Original survivor curves are usually compared with the type survivor curves in the process of determining the probable average service lives from the original data.

2.1 Actuarial Methods

For any actuarial method of life analysis, a complete and aged data record is very essential. A data record is called 'aged' when the property records contain the installation date for each retirement and each survivor.

The statistical compilation and study of human births and deaths have been in practice for a long time. Though the compilation of similar curves for the physical property should have logically followed, it is only since 1902 that such curves have been compiled. The actuarial analysis of

the service lives of depreciable properties is now an established practice in the industries, both regulated and nonregulated.

2.1.1 Survivor Curves

A survivor curve indicates the percentage of the property which survives in service at ages from zero to the maximum life. The actuarial methods involve the treatment of the available aged data to develop the original survivor curve. Usually the original survivor curve will contain some degree of irregularity. Due to these inherent irreqularities, an original survivor curve usually provides insufficient information for depreciation purposes unless some graphical or mathematical standard curves are fitted to smooth and, if necessary, extend them. The process of curve fitting and the Iowa type curves used for the purpose will be discussed in the section following the discussion of the methods of calculating the Original Survivor Curves.

Methods of Calculating the Survivor curves

There are five methods of calculating the survivor curves. These are:

1. Individual Unit method,
2. Annual Rate method,
3. Original Group method,
4. Composite Original Group method, and
5. Multiple Original Group method.

All these methods yield an observed (also called original) life table which is simply a tabulation of the amount of property surviving at each age from zero to the limit of the indicated life. A survivor table is considered two dimensional because it lists percent surviving at varying age intervals. If these original table values are plotted on a graph, the resulting curve is called an original survivor curve. If an original survivor curve does not extend to zero percent surviving, it is called a stubbed curve.

Individual Unit Method

Individual Unit method of calculating the survivor curve is used when only an aged mortality data is available. The data suitable for this method typically consists of only the total number of units retired during a given year together with the age of each unit at retirement. The data record is incomplete in that the plant balance or the number of units surviving in each year is unavailable. The data is illustrated in Table 2.1 [15]. The table shows the retirements arranged in order of their ages. These arranged retirements are then summed from the oldest to the youngest (bottom to top of column 6). From these cumulative retirements, a survivor curve can be plotted as shown Figure 2.2 [15]. It should be noted that the figures in column 9 (Table 2.2) represent only the total retirements during the experience year(s); the data does not reflect other units

Table 2.1 Typical Treatment of Original Data by the Individual-Unit Method [15].

TABLE 2.—TYPICAL TREATMENT OF ORIGINAL DATA BY THE INDIVIDUAL-UNIT METHOD

(Railway pile and frame trestles, 1910 retirements)

Original data as found			Original data organized for calculation of survivor and probable-life curves				Calculation of expectancies and probable life			
Age of trestles removed, years	No. of trestles removed	Trestle-years	Age-interval, years	Original units retired during interval	Survivors of original units at beginning of interval: Units	Survivors of original units at beginning of interval: Percent	Service during interval, %-years	Total remaining in service at beginning of interval, %-years	Expectancy of units in service at beginning of interval, years	Probable life of units in service at beginning of interval, years
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	0	0	0 - 0½	0	190	100.00	50.00	1,094.75	10.95	10.95
1	0	0	0½- 1½	0	190	100.00	100.00	1,044.75	10.45	10.95
2	1	2	1½- 2½	1	190	100.00	99.74	944.75	9.45	10.95
3	0	0	2½- 3½	0	189	99.47	99.47	845.01	8.50	11.00
4	3	12	3½- 4½	3	189	99.47	98.68	745.54	7.50	11.00
5	2	10	4½- 5½	2	186	97.89	97.36	646.86	6.61	11.11
6	1	6	5½- 6½	1	184	96.84	96.58	549.50	5.67	11.17
7	1	7	6½- 7½	1	183	96.31	96.04	452.92	4.70	11.20
8	19	152	7½- 8½	19	182	95.78	90.78	356.88	3.73	11.23
9	31	279	8½- 9½	31	163	85.78	77.63	266.10	3.10	11.60
10	41	410	9½-10½	41	132	69.47	58.68	188.47	2.71	12.21
11	24	264	10½-11½	24	91	47.90	41.59	129.79	2.71	13.21
12	15	180	11½-12½	15	67	35.27	31.32	88.20	2.50	14.00
13	17	221	12½-13½	17	52	27.38	22.91	56.88	2.08	14.58
14	17	238	13½-14½	17	35	18.43	13.95	33.97	1.84	15.34
15	8	120	14½-15½	8	18	9.48	7.38	20.02	2.11	16.61
16	3	48	15½-16½	3	10	5.27	4.48	12.64	2.40	17.90
17	3	51	16½-17½	3	7	3.69	2.90	8.16	2.21	18.71
18	1	18	17½-18½	1	4	2.11	1.84	5.26	2.49	19.99
19	0	0	18½-19½	0	3	1.58	1.58	3.42	2.16	20.66
20	1	20	19½-20½	1	3	1.58	1.32	1.84	1.16	20.66
21	2	42	20½-21½	2	2	1.05	0.52	0.52	0.50	21.00
22	0	0	21½-22½	0	0	0.00	0.00	0.00	0.00	21.50
Total	190	2,080								
Average life		10.947								

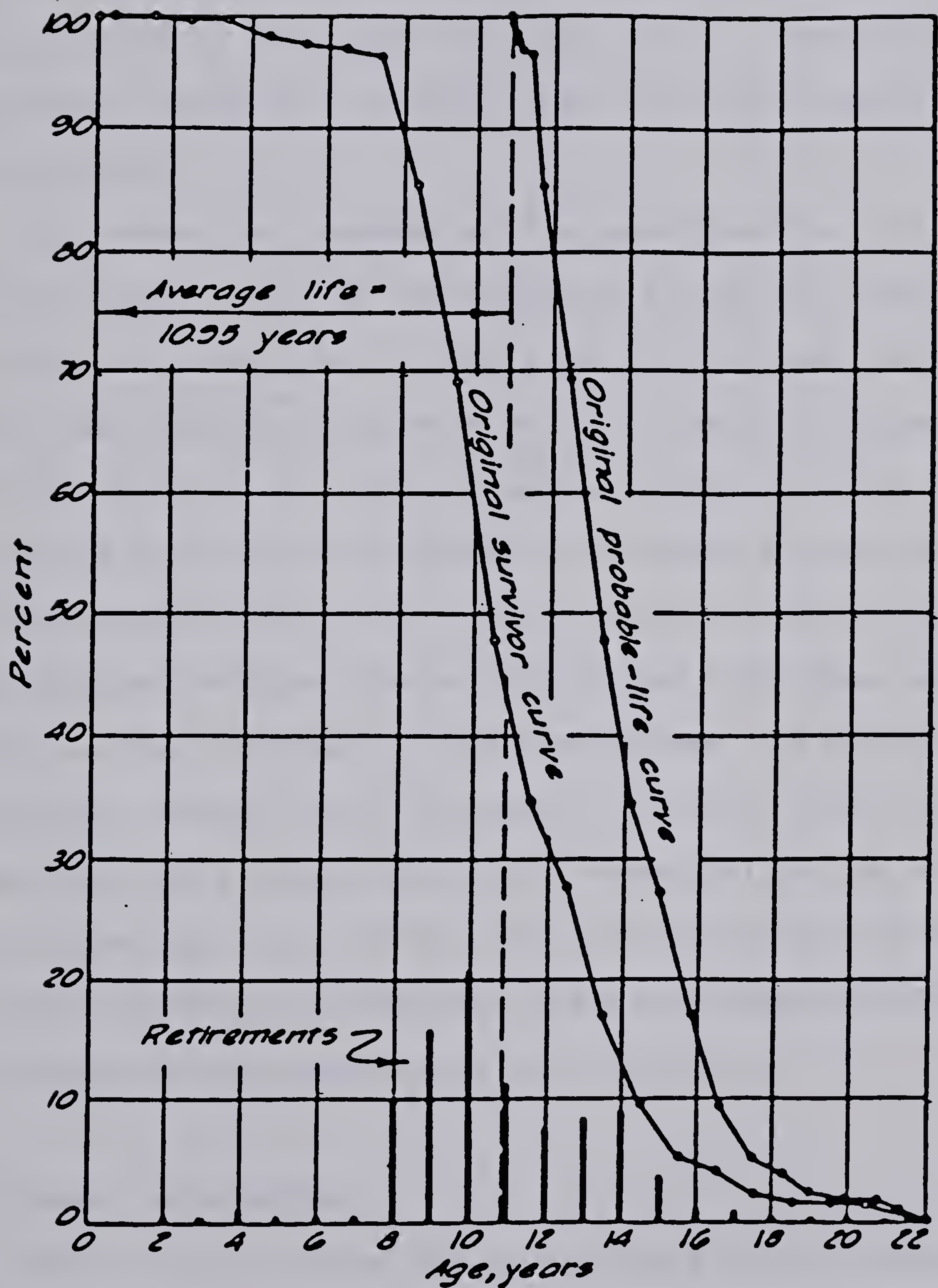


Figure 2.2 Typical Individual-Unit Method Survivor Curve and Probable Life Curve [15]

remaining in service during or at the end of the experience period- every unit has been considered only after its retirement. As such, the method does not take into account other units remaining in service. The average service life is calculated by dividing the total service measured in Unit-Years (Table 2.1, column 3) by the total number of units retired.

Individual Unit Method will approximate the mortality characteristics only if the property account has been in continued existence for a long time and has been maintained at the same number of service units by regular replacements with the units of the same probable average service life. However, since it is infrequent to have an account which has been stationary for a long time, the method should be used with caution. Another limitation of the individual unit method is that it does not give weightage to the units still in service. Hence, for the property accounts that have been in service for a long time, it is essential to use a method which takes into account the units still in service as well as those retired from service. The Annual Rate Method and the Original Group Method are such methods.

Retirement Rate Method

This is also called the Annual Rate Method because the rate of retirement is calculated from each like-age group of units in service during the experience years. As this method considers all the surviving units also, the available data

should show the number of units retired during the period of observation, their respective retirement ages and the number of units in service at the beginning of the observation period and their ages. To quote Winfrey [15]:

> 'The probable average life (obtained by the usual method of determining the area under the completed survivor curve) for survivor curve constructed by the annual rate mehod is a reflection of the average rate of retirement for the observation period chosen. It takes into consideration not only the current retirements ("deaths") but also the units remaining in service (the "living"), and utilizes them in accordance with both their number and age. Such a calculation results in a true picture of rate of retirement since it includes all the prior retirements because the annual rate is dependent upon both the age and the number of units in service.'

It is possible to calculate the annual rates for each age interval from (0-1) to the age of the oldest unit in service if the data (both the retirement and the survivor data) for all the previous vintages upto and including the observation period years is available. From these annual rates, the survival rates can be obtained. A survivor curve results if these calculated survival rates are plotted against the age of the property. The survivor curve so calculated will be stubbed if the earliest vintage is yet to retire completely. In such a case, the stubbed curve has to be extended by the method of curve matching (to be briefly discussed later in this chapter).

The following are the steps involved in calculating a survivor curve:

The first step involves the compilation of the data showing the total number of units (or their total cost) and their respective retirement ages for each year of the observation (or experience) band years. This is illustrated in Table 2.2 [14].

The next step is to compile a similar table for the plant surviving and their respective age for each year of the experience band as shown in Table 2.3 [14].

In the third step, the retirements (Table 2.2) for each experience year from all the vintages as well as the total number of units exposed to retirement (Table 2.3) at the beginning of each age interval are calculated. In Table 2.3, all the units remaining in service on the diagonal steps are from the same age interval.

Step 4 involves the calculation of the retirement rates for each age interval from the retirements and balances as calculated in the previous step. Once these retirement rates are calculated, the corresponding year end survival rates can be obtained by the product of the begining of the year survivor rate and (1 retirement rate for the year under consideration).

The next step is to plot the survivor and the retirement rates as far as they extend as indicated in Figure 2.3 [14].

The fifth step involves the smoothing of the original survivor curve so obtained (and if required, extending it also) by either a mathematical or a graphical curve fitting

Table 2.2 Typical Treatment of the Retirement Data by the Retirement Rate Method [14].

TABLE 7.3. DOLLAR COST OF CENTRIFUGAL GAS PUMPS RETIRED EACH YEAR 1940 TO 1950
Arrangement of Retirements for Calculation of Average Service Life by the Retirement Rate Method

Year	Dollars cost installed during year	Dollars original cost retired during the calendar year										
		1940	1941	1942	1943	1944	1945	1946	1947	1948	1949	1950
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1919	1,578	......	.....	......	..	......	316	......	......	......	......	......
1920	3,260	......	1,512	202	..	......	812	......	......	......	......	......
1921	5,980	......	.....	......	..	......	......	......	2,115	......	......	......
1922	61,930	10,604	.....	......	..	......	......	......	......	......	......	......
1923	24,888	......	.....	......	..	......	......	......	23,411	......	......	......
1924	54,680	365	.....	......	..	......	......	......	8,612	......	......	11,712
1925	......	......	.....	......	..	......	......	......	......	......	......	......
1926	60,980	......	.....	24,116	..	19,100	......	8,200	......	......	......	......
1927	50,600	8,206	.....	412	..	......	......	......	......	8,412	19,815	......
1928	129,612	6,055	.....	5,812	..	7,381	......	17,000	17,641	12,200	24,452	......
1929	86,412	......	4,612	......	..	......	......	......	2,792	......	......	......
1930	60,812	......	.....	......	..	......	24,280	......	......	......	......	17,601
1931	8,916	......	.....	......	..	......	......	......	......	......	......	......
1932	22,102	......	.....	......	..	......	2,402	......	7,465	......	9,224	......
1933	......	......	.....	......	..	......	......	......	......	......	......	......
1934	......	......	.....	......	..	......	......	......	......	......	......	......
1935	80,916	......	.....	......	..	......	......	10,609	......	......	......	6,129
1936	3,012	......	.....	......	..	......	2,016	......	......	......	......	......
1937	901	......	.....	......	..	......	......	......	901	......	......	......
1938	1,206	......	.....	244	..	......	......	......	......	......	......	......
1939	3,600	......	401	......	..	......	......	......	......	......	......	689
1940	15,215	......	.....	......	..	......	......	1,261	......	......	......	......
1941	20,606		.....	......	..	877	......	1,060	......	......	......	2,141
1942	712			......	..	......	......	......	......	......	......	712
1943	......				..	......	......	......	......	......	......	......
1944	......					......	......	......	......	......	......	......
1945	......						......	......	......	......	......	......
1946	44,900							......	916	......	......	......
1947	91,560								......	......	......	......
1948	180,111									......	......	......
1949	102,434										618	......
1950	90,676											......
Total.	1,207,599	25,230	6,525	30,786	0	27,358	29,826	38,130	63,853	20,612	54,109	38,984

Table 2.3 Typical Treatment of the Survivor Data by the Retirement Rate Method [14].

TABLE 7.4. DOLLAR COST OF CENTRIFUGAL GAS PUMPS IN SERVICE JAN. 1, 1940, TO JAN. 1, 1950
Arrangement of Property in Service for Calculation of Average Service Life by the Retirement Rate Method

Year	Dollars cost installed during year	Dollars remaining in service Jan. 1 of year										
		1940	1941	1942	1943	1944	1945	1946	1947	1948	1949	1950*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1919	1,578	1,578	1,578	1,578	1,578	1,578	1,578	1,262	1,262	1,262	1,262	1,262
1920	3,260	3,260	3,260	1,748	1,546	1,546	1,546	734	734	734	734	734
1921	5,980	5,980	5,980	5,980	5,980	5,980	5,980	5,980	5,980	3,865	3,865	3,865
1922	61,930	24,989	14,385	14,385	14,385	14,385	14,385	14,385	14,385	14,385	14,385	14,385
1923	24,888	24,888	24,888	24,888	24,888	24,888	24,888	24,888	24,888	1,477	1,477	1,477
1924	54,680	20,689	20,324	20,324	20,324	20,324	20,324	20,324	20,324	11,712	11,712	11,712
1925	......	....	....	....	....	....	....	....	....	....	....	....
1926	60,980	51,416	51,416	51,416	27,300	27,300	8,200	8,200	....	....	....	....
1927	50,600	50,600	42,394	42,394	41,982	41,982	41,982	41,982	41,982	41,982	33,570	13,755
1928	129,612	118,855	112,800	112,800	106,988	106,988	99,607	99,607	82,607	64,966	52,766	28,314
1929	86,412	70,812	70,812	66,200	66,200	66,200	66,200	66,200	66,200	63,408	63,408	63,408
1930	60,812	58,690	58,690	58,690	58,690	58,690	58,690	34,410	34,410	34,410	34,410	34,410
1931	8,916	8,916	8,916	8,916	8,916	8,916	8,916	8,916	8,916	8,916	8,916	8,916
1932	22,102	20,467	20,467	20,467	20,467	20,467	20,467	18,065	18,065	10,600	10,600	1,376
1933	......	....	....	....	....	....	....	....	....	....	....	....
1934	......	....	....	....	....	....	....	....	....	....	....	....
1935	80,916	80,406	80,406	80,406	80,406	80,406	80,406	80,406	69,797	69,797	69,797	69,797
1936	3,012	3,012	3,012	3,012	3,012	3,012	3,012	996	996	996	996	996
1937	901	901	901	901	901	901	901	901	901	....	....	....
1938	1,206	1,206	1,206	1,206	962	962	962	962	962	962	962	962
1939	3,600	3,600	3,600	3,199	3,199	3,199	3,199	3,199	3,199	3,199	3,199	3,199
1940	15,215		15,215	15,215	15,215	15,215	15,215	15,215	13,954	13,954	13,954	13,954
1941	20,606			20,606	20,606	20,606	19,729	19,729	18,669	18,669	18,669	18,669
1942	712				712	712	712	712	712	712	712	712
1943	......					....	....	....	....	....	....	....
1944	......						....	....	....	....	....	....
1945	......							....	....	....	....	....
1946	44,900								44,900	43,984	43,984	43,984
1947	91,560									91,560	91,560	91,560
1948	180,111										180,111	180,111
1949	102,434											101,816
1950	90,676											
Total	1,207,599	550,265	540,250	554,331	524,257	524,257	496,899	467,073	473,843	501,550	661,049	709,374

* Since 1950 is the last year for which the retirements are used in the experience band, the property in service Jan. 1, 1951, is not used in the retirement rate analysis for the experience band of 1940–1950.

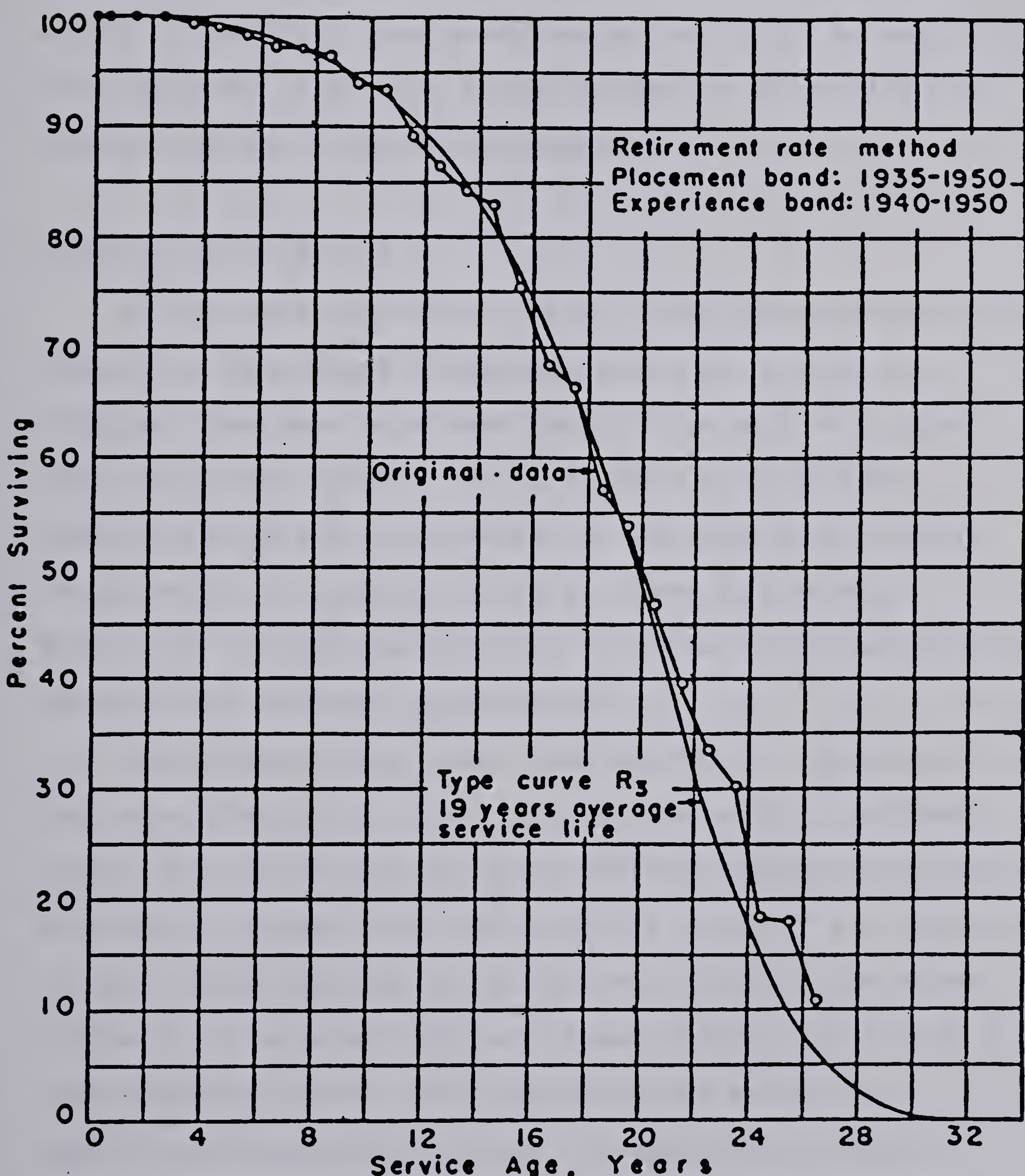


Figure 2.3 Typical Original Survivor Curve Developed by Retirement Rate Method [14].

process (to be discussed later).

The last step is to determine the average service life from the total area under the survivor curve. The total area under the survivor curve is representative of the total amount of property considered (which is 100%). As such, the area measured in percent-years divided by 100 yields the average service life of the property.

Original Group Method

As the name indicates, the original group method is the process of developing a survivor curve for a specific vintage. Sometimes more than one vintage will be grouped together for the study. Such a process is called the Composite Original Group Method or the Multiple Original Group method (in actual practice, there is a subtle difference between the Composite Original Group Method and the Multiple Original Group Method).

The vintage group under consideration is observed to determine the percent surviving and the percent retired. Using this, a survivor curve is plotted. The Original Group Method will result in a stub survivor curve if all the units in the vintage are yet to be retired. However, the curve gradually extends all the way to zero percent surviving with the progress of time. The Original Group Method of calculating the survivor curve, if applied to develop individual survivor curves of successive vintage groups, is useful in determining any trends in the average service life

and the mortality distribution pattern.

Curve Fitting to the Original Survivor Curves

If the developed original survivor curve is found to be stubbed and/or scattered, it becomes essential to first extend and/or smooth the original curve using one of the curve fitting methods. It is possible in practice to fit a number of different curves to the original mortality data. The methods used for the purpose are:

1. Iowa Type Curves,
2. Gompertz-Makeham Distribution Curves,
3. h - curves or the Kimball Curves,
4. polynomials, and
5. the Weibull Distribution.

Of the five types mentioned above, the first three systems of curves are used more than the last two methods. However, Iowa type curves are the most popular curve types used for life analysis.

Henderson [6] of the Iowa State University conducted tests to compare the performance of these five methods of curve fitting and found that

1. though the graphical methods have only a finite set of curves to fit and appear to be more of judgement oriented methods, it was found that they perform very well when used in conjunction with a computer for the curve fitting procees (ie when a large number of calculations are made),

2. the five actuarial methods of curve fitting are not significantly different when tested with simulated data. However, based on the results of curve fitting of actual property data, Iowa and h - types of curves were found to be superior to the other methods.

As already mentioned, the Iowa Type Curves are the most extensively used type curves for this purpose. Extensive research has been conducted to determine their validity. As such, they will be discussed in greater detail now.

Iowa Type Standard Curves

The research and development of the Iowa type curves were started around 1921 at the Iowa State College. The first publication of this research (Bulletin 103, 1931) contained 7 types of Iowa curves. Subsequent research has enabled the development of more types of Iowa survivor curves. At present there are a total of 31 type curves which are classified into 4 main groups namely, left modal curves, symmetrical modal curves, right modal curves and the origin modal curves.

Figures 2.4 through 2.7 [11] illustrate the four main groups mentioned above. As seen from the figures, the abscissa values are expressed in percent of the average service life while the ordinate values are expressed in percent surviving. Each curve type is designated by a combination of a letter and a number indicative of the

1. degree of symmetry, and

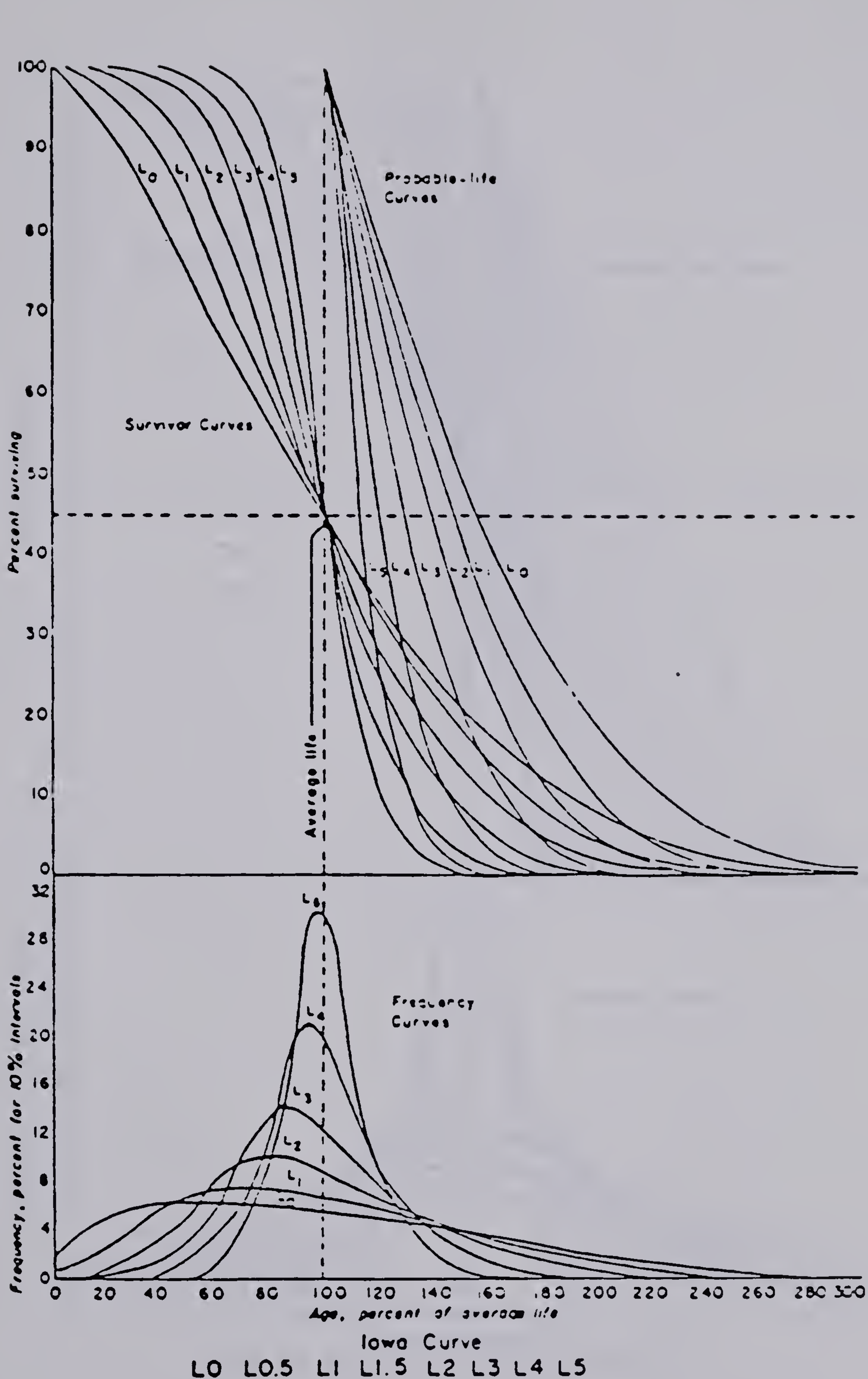


Figure 2.4 Iowa Type Left Modal Curves [11].

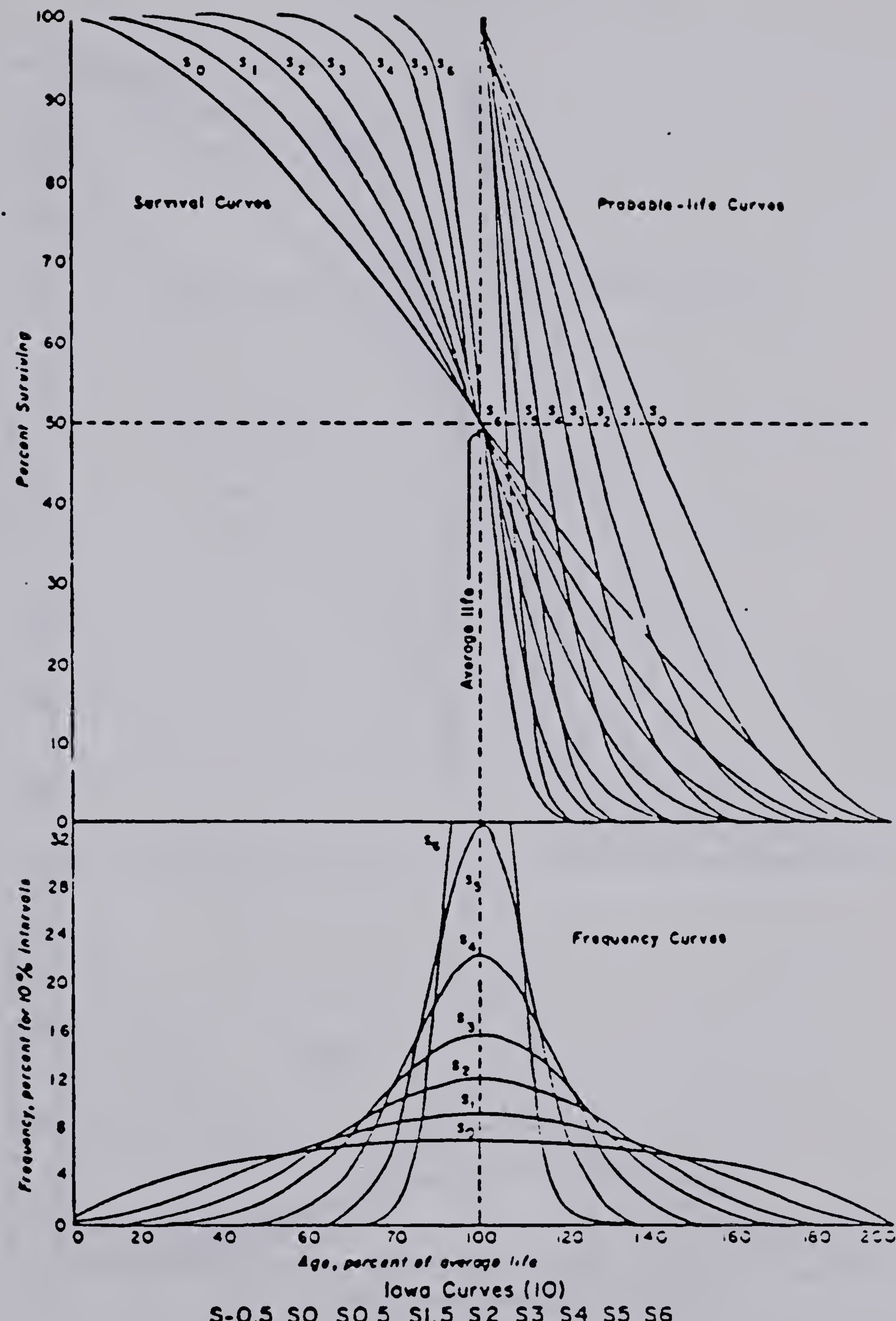


Figure 2.5 Iowa Type Symmetrical Modal Curves [11].

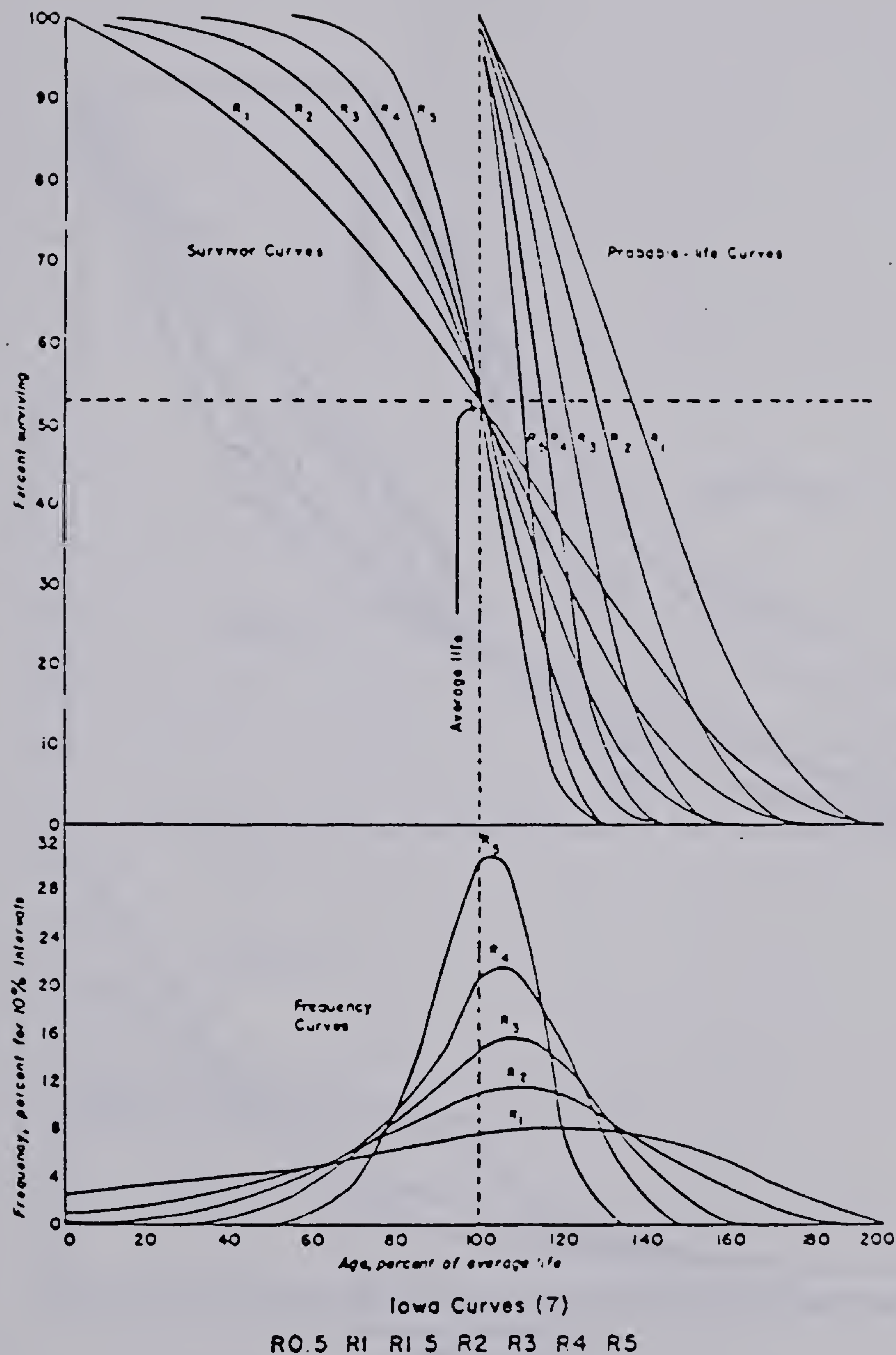


Figure 2.6 Iowa Type Right Modal Curves [11].

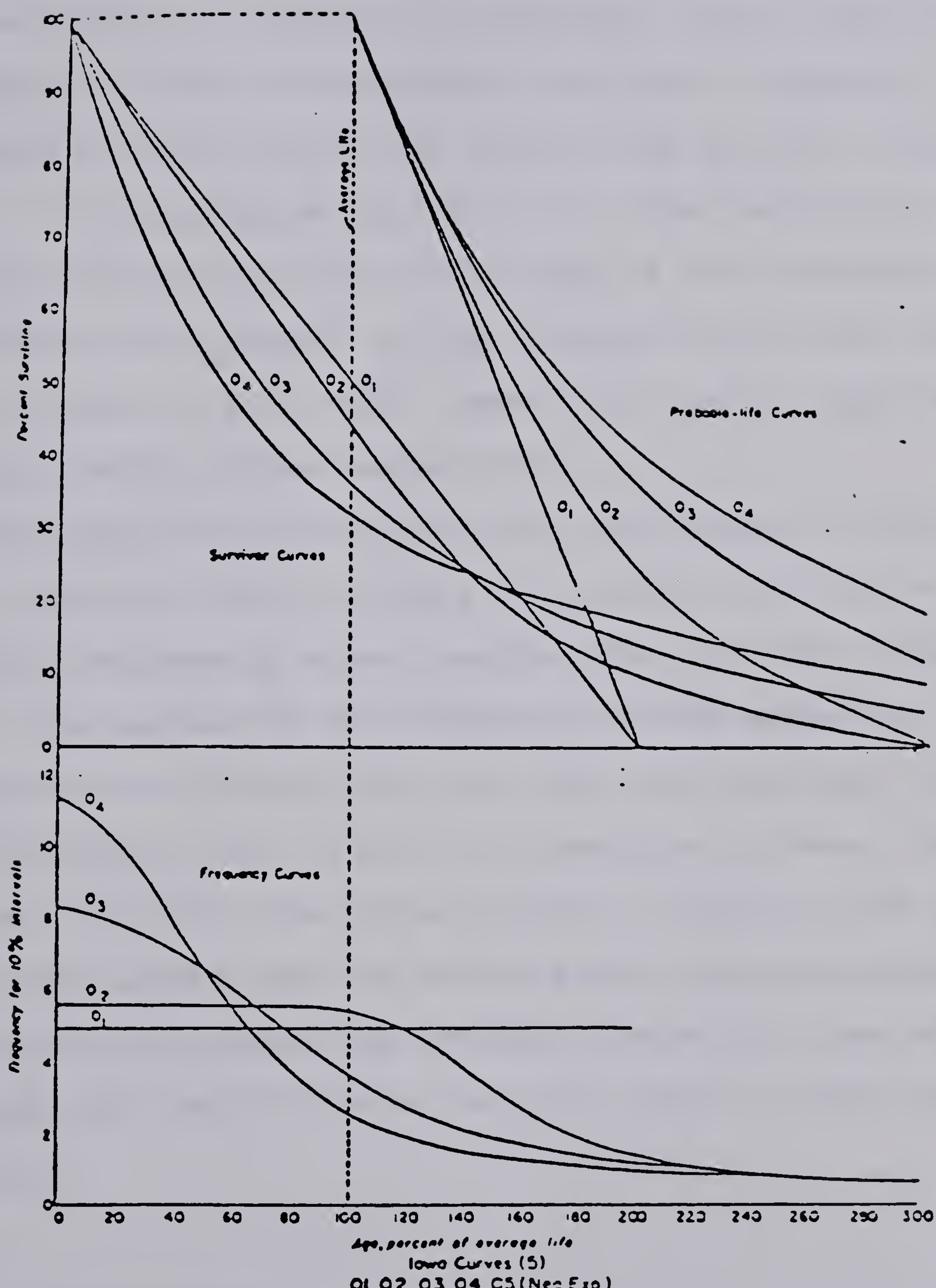


Figure 2.7 Iowa Type Origin Modal Curves [11].

2. degree of peakedness, respectively.

Degree of Symmetry

The degree of symmetry of a survivor curve refers to the symmetry of the corresponding retirement frequency distribution curve rather than that of the survivor curve itself. It is designated by L,S,R or O. The designation is indicative of the position of the mode of the retirement distribution with respect to the average service life. The letters stand for left modal, symmetrical modal, right modal and origin modal curves respectively.

The position of the modal value with respect to the average service life determines the percent surviving at any time for the property under consideration. For left modal curves, the percent of the property retiring before the average service life is more than that retiring after the average service life. In case of symmetrical curves, they are identical while the latter percent is more in case of right modal curves. All the origin modal curves have more percent retiring before the average service life than after it except for the 'O1' curve for which the two values are identical.

Degree of Peakedness

The peakedness of the frequency distribution curve is indicated by the number assigned to the type curves. The

range of this number for the different groups is as follows:

CURVE TYPE	RANGE	NUMBER OF CURVES IN THE GROUP
L	0 to 5	8
S	-0.5 to 6	10
R	0.5 to 5	8
O	1 to 5	5

These numbers indicate the relative height of the mode. A larger number indicates a steeper curve. However, these numbers are not absolute values but are relative within the respective mode group classification. The larger the assigned number of a curve, the higher will be its mode (relative to the curves with smaller numbers within that modal group only).

The area under any survivor curve is measured in percent-years. Consequently, the total area under a survivor curve divided by 100 gives the average service life of the property under consideration.

2.2 Semiactuarial Methods

These methods are also called the turnover methods. There are many semiactuarial methods of life analysis. These methods of life analysis are applicable only if the

available data is not aged (ie., if the ages of the retired units as well as the surviving units are unknown). Due to this restraint, the turnover methods rely on the ratios of the total annual retirements to the total annual plant balances without regard to the ages of the property, either retired or surviving. These methods yield an estimation of the probable average service life only, without any indication of the probable distribution of retiremnts.

The three turnover methods of life analysis are:

1. Turnover Period method,
2. Half-Cycle Ratio method, and
3. Asymptotic method.

2.2.1 Turnover Period Method

The data records necessary for this method should contain the annual additions, annual retirements and the annual plant balances. The 'turnover period', which is the time required to turnover the plant balance, is calculated as follows:

If the plant balance at the beginning of a period is 'X', and if the cumulative plant retirements over 'n' periods (usually years) amounts to 'X', then the turnover period is 'n' years. The turnover period so calculated is not equal to the average service life, but is only an indication of the same.

A few subtle variations of the method are also in existence. In one of the variations the period between

identical values of the cumulative retirements and the cumulatinve additions is the turnover period. In yet another variation, the period between the identical values of the plant balance and the cumulative plant additions is the turnover period. The turnover period method is applicable to nongrowing accounts only.

2.2.2 Half-Cycle Ratio Method

This method is applicable even when the data available is only half as much as that required for the Turnover Period Method. This is essentially an iterative method. Under this method, a turnover period (N) is assumed. A year (X) is chosen for which the total retirements are known (Rx). The plant balance (B) for a year N/2 periods away is obtained from the records. This plant balance 'B' is divided by 'Rx' to obtain the calculated turnover period 'Nc'. The procedure is repeated till 'Nc' and 'N' correspond with one another. As in the case of the turnover period method, the turnover period obtained has to be adjusted for growth and retirement dispersion in order to get an estimation of the average service life.

2.2.3 Asymptotic Method

This method is dependent on the fact that any continuous property group maintained at the full operating level through replacements (ie. through regular plant additions as required) will reach limiting ratios of annual

additions and retirements to the plant balance. In this method, the average service life is the reciprocal of the geometric mean of the additions and retirements ratios.

The limiting asymptotes are determined by plotting the curves of the two ratios and fitting them to a curve with the equation:

$$y = a + (b/x) + (c/x^2)$$

where y is either the addition ratio or the retirement ratio, x is the year or age scale and a,b,c are constants.

Since the actuarial methods are by far the best, it is advisible to use them rather than the semiactuarial methods if the required type of data is available. The turnover methods do not yield good results if the account is young, if there are a large number of retirements or if the additions and retirements are not reasonably uniform. Another drawback of the semiactuarial methods is that they do not yield an estimation of the retirement distribution pattern.

2.3 Simulation Methods

Simulation methods are superior to semiactuarial methods because they yield estimates of both the average service life and the retirement distribution pattern. However, if suitable data is available, actuarial methods are better than the simulation methods.

There are three important simulation methods of life analysis. They are:

1. Simulated Plant Record Method,
2. Computed Mortality Method, and
3. Transparent Plant Balance Method.

Of the three methods mentioned above, the first two will be discussed in this chapter. The immediately following chapter contains a detailed review of the Transparent Plant Balance Method. The basis for all of these simulation methods is the Iowa type curves.

2.3.1 Simulated Plant Record Method

This method was first developed and proposed by Bauhan [1].

> 'The Simulated Plant-record method of life analysis consists of applying such (Iowa Standard Curves) standard mortality dispersions to the record of plant additions and discovering by trial and error which particular combination of average life and mortality dispersion (sometimes hereinafter called a mortality pattern or a mortality characteristic) best simulates in calculated results the record of actual balances or actual retirements. The method serves equally whether applied to records of balances or retirements,...'

One of the requirements of this method is the availability of suitable unaged data; annual plant additions, annual plant balances and annual retirements. Such data should be available for all the years during which the account has been in existence.

The analyst must specify a range of the average service lives over which the trials should be conducted. It is also

necessary to specify the standard curve types to be tested. The plant balances are simulated by the successive application of the survival rates (derived from the specified standard type curve) to each of the vintage additions. Now, the plant balance for each year is calculated by the addition of the individual plant balances for that year due to all the earlier vintages. Table 2.4 illustrates this method for the first 5 years of an account where the actual plant additions for the five years are 100,000, 5,147, 11,074, 9,149 and 11,737 units respectively (column 2). The actual plant balances are 98,961, 101,986, 105,168, 108,424 and 111,326 units respectively (bottom row). An average service life of 10 years and L0 type curve have been specified for the test. For this L0-10 curve type, the cumulative retirement rates for the first five years are 1.1063%, 5.0783%, 10.2173%, 16.0068% and 22.1395% respectively. Therefore the corresponding survival rates are 98.8937%, 94.9217%, 89.7827%, 83.9932% and 77.8605%. These rates are applied to the first vintage (100,000 units) to calculate the plant balances in each of the five years due to that vintage. For example,

Plant balance in the 2nd year due to the first vintage

$$= \text{(vintage size)} \times \text{(survival rate)}$$

$$= (100{,}000) \times (94.9217)/100$$

$$= 94{,}923 \text{ units.}$$

Table 2.4 Simulation Process for the SPR Method

		Yearly Vintage Plant Balances				
Vintage Year	Vintage Size	1	2	3	4	5
1	100,000	98,894	94,923	89,783	83,993	77,861
2	5,147	-	5,090	4,886	4,621	4,323
3	11,074	-	-	10,951	10,512	9,943
4	9,149	-	-	-	9,048	8,684
5	11,737	-	-	-	-	11,607
Simulated Plant Balances		98,894	100,013	105,620	108,174	112,418
Actual Plant Balances		98,161	101,986	105,168	108,424	111,326

Only the first four rates are applied to the second year vintage because it was installed in the second year and was only 4 years old at the completion of the fifth year of the account. Now, the plant balances from all the vintages for each year are added to produce the simulated plant balances (2nd row from the bottom). This set of simulated plant balances will be compared to the actual plant balances to determine the quality of the simulation. In the illustration provided in Table 2.4, the calculations are shown for only the first five years of the account. However, in real practice, this will be done for the entire experience band. This process will be repeated for all the possible combinations of the average service life and the Iowa Type Curves and the best combination will be selected.

In this method of life analysis, an index is used as a measure of the closeness of fit between the actual and the simulated plant balances. The index being used is the Conformance Index which is given by Equation 2.1.

$$CI = \frac{\sum_{j=1}^{n} B_j / n}{\left[\sum_{j=1}^{n} (B_j - B'_j)^2 / n\right]^{1/2}} \qquad \text{Eqn. 2.1}$$

where,

n = number of years in the experience band,

B_j = actual plant balance in the jth year of the experience band,

B'_j = simulated plant balance in the jth year of the experience band, = $\sum_{i=1}^{j} (N_i) x (S_{ij})$

N_i = plant additions in the ith year of the account,

S_{ij} = survival rate in the jth year for the property installed in the ith year of the account. This value is obtained from the Iowa type curve.

An arbitrary range for grading the curves has been specified: (0 to 25)-poor; (25 to 50)-fair; (50 to 75)-good; (75 to infinity)-excellent.

It is obvious from the above discussion that for the SPR method to be used, the unaged data record should be available for all the years from the beginning of the account. However, if some of the initial data is unavailable, a historical trending is done for the years in which the data is missing. This trending is done by generating the missing data using the available data for the later years as the basis. The trending is more one of subjective judgement than one of an empirical nature. This is likely to cause some errors in the final results especially if a large portion of the earlier data is missing from the records.

The performance of the SPR method has been extensively studied at both the Iowa State University and the Western Michigan University. It has been found that the SPR method reflects the life characteristics of the property which has nearly the same life characteristics for all the vintages [4]. However the scale specified to grade the quality of the fit was found to be arbitrary. According to Fitch et al [4],

'Reservations about the usefulness of the indexes

> measuring the matching of annual balances and the length of curve stub also arise......In 1947, Bauhan presented a scale of ratios for the calculated CI based on the empirical evidence at the time; but with more evidence and a different technology today, it is not necessarily applicable.'

2.3.2 Computed Mortality Method

This is a recent method of life analysis developed to overcome the limitations of the SPR method of life analysis in cases of incomplete data availability. However, this method is still in the developmental stages.

The simulation is usually begun in a specified experience year for which the retirements, plant additions and plant balances are known. The first step is to estimate the vintage composition of the beginning of the year plant balance for the year under consideration. This process is again subjective and is dependent on the personal experience and expertise of the analyst. From these estimated vintage survivors, retirements by vintage for that year are simulated according to an assumed survivor curve and average service life. All these individual vintage retirements are summed. The average service life is varied till the simulated retirements match the total actual retirements for that year. The end of the year survivors from each vintage are calculated and appended to the data matrix. This procedure is repeated for each year. The average service life used for each year is recorded. Thus, the method is likely to yield a different average service life for each year of the experience band.

The method appears to need no indices to measure the closeness of fit of the annual retirements since each year it must duplicate a single retirement total. The choice of the curve type used to simulate the retirements is subjective. The method by which a property life estimate is made from the Computed Mortality data is yet to be well defined. One method uses the trend of the recorded curve averages used to calculate the retirements. Alternately, the full matrix of simulated aged data may be used to calculate vintage group depreciation or may be analyzed by actuarial methods to estimate a life to be used with broad group depreciation. One other approach uses time series analysis to forecast a trend of the average service life. However, the performance of the model is yet to be studied in detail.

The following chapter contains a detailed discussion on the Transparent Plant Balance Method of life analysis.

3. TRANSPARENT PLANT BALANCE METHOD

The Transparent Plant Balance Method of life analysis has been derived from the Simulated Plant Record Method (SPR). Though the SPR is quite useful as a method of life anlysis when unaged and complete data is available, the performance of the method becomes unsatisfactory if only partial data is available. This is because of the method employed in the SPR for the generation of the missing data. Once this missing data is generated, it becomes a permanent part of the available data record. Using this generated data in conjunction with the actual data, the SPR method selects the best fitting curve and the average service life. Consequently, it is quite likely that the selected mortality characteristics are not representative of the true mortality characteristics (see the previous chapter for details about the SPR method).

The Transparent Plant Balance Method was developed by Edmonton Telephones to reduce the subjectivity of the SPR method in generating the missing data. As such, the Transparent Plant Balance Method (TPBM) is applicable only to the cases where the available data is inadequate for the SPR method to be used. The TPBM employs an iterative method of generating the missing data; it is generated by using various combinations of the mortality characteristics. The charactericstics of the generated data which produces the simulated plant balances closest to the actual plant balances are selected as the true mortality characteristics

of the data set. Thus, the procedure adopted by this method is such that the mortality characteristics used to generate the missing data are identical to the finally selected mortality characteristics of the property under consideration.

3.1 Terminology

The following are a few of the definitions associated with the TPBM and are illustrated on Figure 3.1.

<u>Transparent Band</u> is the band comprised of the years for which the actual data is not available. As a result of this, the missing data will be generated with some assumed characteristics.

<u>Observation Band</u> is comprised of the years for which the actual data is available.

<u>Growth Profile</u> is the profile of the curve obtained by plotting the plant additions against time (years-representing the age of the account). The growth profile is exponential if the curve obtained by plotting the plant additions is increasing with years in an exponential manner.

3.2 Process of the TPBM

The TPBM is applied in four stages namely,

1. selection of the initial condition,
2. generation of the data,
3. initial selection, and

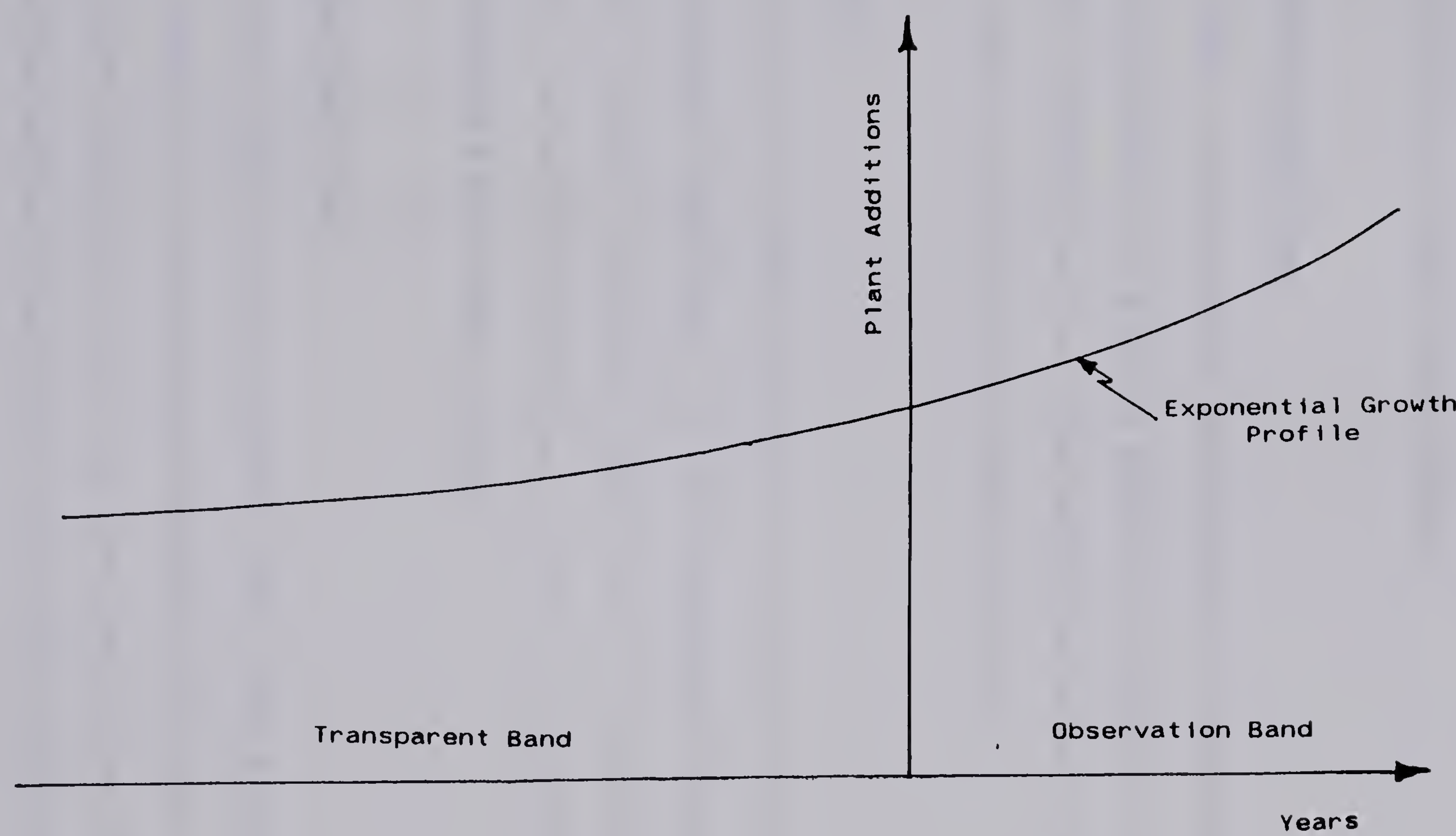


Figure 3.1 Typical Growth Profile and the Associated Definiitions of the TPBM

4. final selection.

3.2.1 Initial Condition

It is first necessary to assume some initial conditions for the analysis as described below:

Type of Growth Profile

The TPBM, as developed by Edmonton Telephones, always assumes an exponential growth profile (Figure 3.1) for the annual plant additions (ie. the model assumes that the plant additions grow annually by a fixed percent).

Growth Rates

Given that the TPBM assumes an exponential growth of the annual plant additions, it is necessary for the analyst to specify a range of growth rates and an incremental value (incremental value is the value by which the growth rate is to be incremented for each trial).

Average Service Life

As in the case of the growth rate, the analyst must provide a range for the average service life over which the test has to be conducted. During the test, the TPBM increments the average service life (ASL) from the intial to the final value in steps of one year.

Curve Type

Usually the analyst specifies all the 31 Iowa type curves for the analysis unless there are some strong reasons to believe that the actual mortality characteristic curve is not represented by a specific standard curve.

Transparent Band Length

Although the length of the Transparent Band will be usually a known parameter, occasionally the analyst might come across a situation when the length of the Transparent Band (ie. the age of the account) is not known definitely; only an approximate length of the band might be known. If this is the case, the analyst will have to specify an estimated length of the Transparent Band. If the Transparent Band (TB) length is known, which usually will be the case, the known length has to be specified.

3.2.2 Data Generation

Using the initial conditions as specified by the analyst, the TPBM generates the missing data in the Transparent Band. This generated data will be of the unaged type comprised of the annual plant additions and annual plant balances.

As mentioned earlier, the TPBM assumes a compounding growth for the plant additions. To begin with, the plant additions for the transparent band will be generated using the specified growth rate as shown below:

$$N_{i-1} = N_i/c \text{ for } i = k+1 \text{ to } 2. \qquad \text{Eqn. 3.1}$$

where 'k' is the length of the Transparent Band,
'N_i' is the plant additions for the ith year of the Transparent Band,
'c' is the compounding factor (for example, 1.07 for 7% growth rate etc.)

Thus, for example, if the Transparent Band length is 15 years, the Observation Band length is 7 years and the growth rate specified is 5%, then the plant addition for the last year (15th year) of the Transparent Band is given by dividing the actual plant additions for the first year of the Observation Band by the specified exponential factor (1.05 in this case). The plant addition for the 14th year of the Transparent Band is obtained by dividing the plant balance for the 15th year (calculated as described above) by the specified exponential factor of 1.05. This procedure is repeated to get the calculated plant additions for each year of the Transparent Band. Once the plant additions for the entire Transparent Band is generated, the problem is somewhat similar to the SPR method of life analysis. In the Transparernt Plant Balance Method of Life Analysis, the simulated plant balances will be matched to the actual plant balances in the Observation Band to determine the best combination of the characteristics.

After the plant additions are generated for the Transparent Band using the initial growth rate and the other

parameters, the plant balance for the Observation Band are simulated for each possible combination of the specified average service life and the Iowa type curve. The simulation process is similar to that of the SPR method (refer to Table 2.4). For example, if the specified range of the average service life is 8 to 12 years in steps of 1 year each, the curves to be tested are all the 31 Iowa type curve and if the specified range of the growth rate is from 1.0 to 1.2 the incremental steps of 0.01, the TPBM would generate and test a total of (31)x(5)x20=3,100 sets of data. To give a better understanding of the process, the nested DO loops used in the computer program are diagrammatically shown in Figure 3.2.

3.2.3 Initial Selection

It is obvious from the preceding discussion on data generation that it is advantageous, if possible, to eliminate some of the generated data sets by some process of initial selection. Hence, the TPBM employs an initial selection process wherein many of the improbable data sets are eliminated at the beginning. The process of the intial selection used in the TPBM is discussed in the following few paragraphs.

The criterion used for the initial selection is the accuracy of the simulated plant balance for the first year of the Observation Band. From the generated plant additions for the Transparent Band, it is possible to simulate the

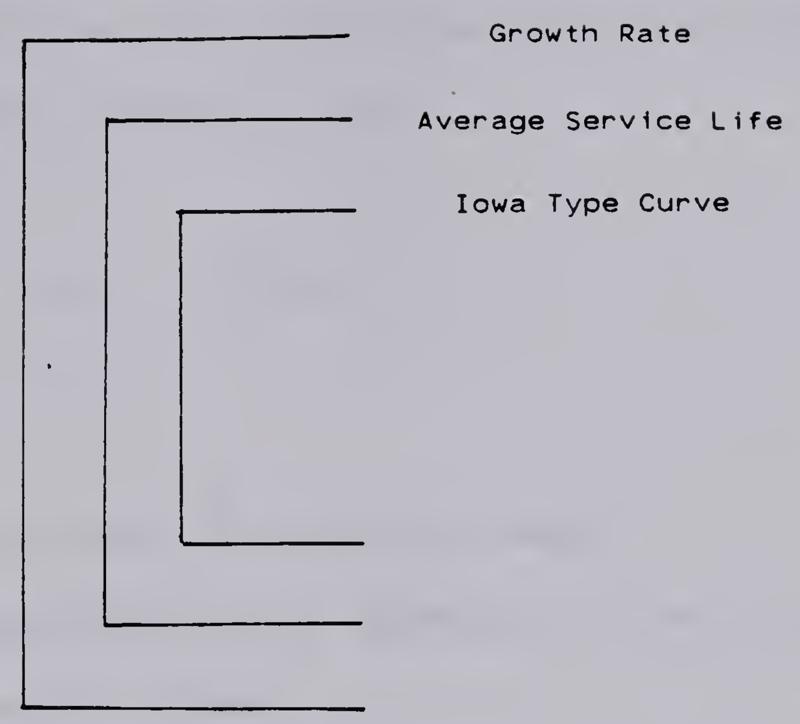


Figure 3.2 Simulation Process of the TPBM.

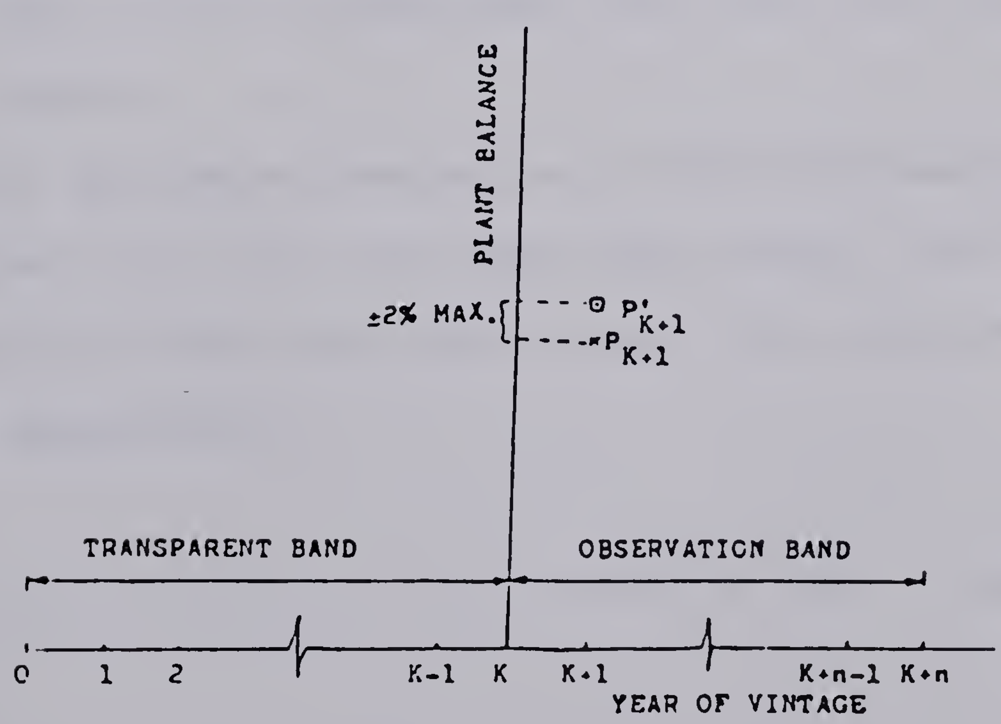


Figure 3.3 Initial Selection Process of the TPBM [11].

plant balance for the first year of the Observation Band using the survival rate derived from the Iowa Type Curve and the average service life under consideration. This process is mathematically expressed as,

$$P'_{k+1} = \sum_{i}^{k+1} N_i S_{i,k+1}$$

where,

k = length of the Transparent Band,

P'_{k+1} = simulated plant balance in the first year of the Observation Band,

N_i = generated plant additions in the ith year of the Transparent Band,

$S_{i,k+1}$ = % surviving in the (k+1)th year from the vintage installed in the ith year of the Transparent Band. This value is obtained from the Iowa type survivor curve.

Thus, for each combination of the initial parametrs (ASL, growth rate and the Iowa type curve), the P'_{k+1} is calculated as described above. Now, the criterion for the initial selection is

$$| P_{k+1} - P'_{k+1} | / P_{k+1} \leq 0.02 \text{ or } 2\% \qquad \text{Eqn. 3.2}$$

where,

P_{k+1} = actual plant balance in the 1st year of the Observation Band.

Hence the criterion for the selection is that the relative error of the simulated plant balance for the first year of the Observation Band should be less than or equal to 2% of the actual plant balance. This figure of 2% is an arbitrarily selected value and does not have any empirical substantiation. This process of initial selection is illustrated in Figure 3.3 [11].

3.2.4 Final Selection

In this phase of the analysis, the initially selected combinations of the mortality characteristics (ie. combination of the ASL, Iowa type curve and the growth rate) are tested for the final selection. The combination of the characteristics that produces the best matching simulated plant balances in the Observation Band is selected as the characteristics of the data under consideration. As in the case of the SPR method of life analysis, the Conformance Index developed by Bauhan [1] is used as an index of the closeness of fit. The Conformance Index (CI) is based on the minimum sum of the squares criterion. The CI gives an indication about which set of the simulated plant balances has the least sum of the squares of the differences with the actual plant balances.

We have,

$$CI = \frac{\text{Average of the actual plant balances in the comparison years}}{\text{Root Mean Squared Deviation Between the simulated and the actual plant balance.}}$$

Mathematically, the CI can be defined as,

$$CI = \frac{\sum_{j=1}^{n} P_j / n}{[\sum_{j=1}^{n} (P_j - P'_j)^2 / n]^{1/2}} \qquad \text{Eqn. 3.3}$$

where,

n = number of years in the Observation Band,

P_j = actual plant balance in the jth year of the Observation Band,

P'_j = simulated plant balance in the jth year of the Observation Band, $= \sum_{i=1}^{j} (N_i) x (S_{ij})$

N_i = plant additions in the ith year of the account,

S_{ij} = survival rate in the jth year of the property installed in the ith year of the account. This value is obtained from the Iowa type curve.

An arbitrary scale for grading the curve has been established:

0 to 25	POOR
25 to 50	FAIR
50 to 75	GOOD
75 to INF	EXCELLENT

3.3 Performance of the TPBM

A performance evaluation (sensitivity analysis) of the TPBM was conducted by Tharumarajah [11]. A summary of the investigation is provided in this section.

The objectives of the study were:

1. to examine the effects of the length of the transparent band used on the final results,
2. to determine the minimum actual data required for the TPBM to produce acceptable results,
3. to test the validity of the compounding growth rate assumption used, and
4. to investigate the adequacy of the Conformance Index to indicate the correct mortality characteristics.

Transparent Band Length

Since the length of the Transparent Band will be occasionally an unknown factor, the effect of any error in the specification of the Transparent Band length on the finally selected curve was investigated.

Minimum Actual Data Requirement

The virtue of the method is that it is applicable to cases where sufficient data is not available to use any other standard method. However, even for the TPBM to be used, there is some minimum actual data requirement in order to derive meaningful results. Hence the performance of the model for varying lengths of the Observation Band was evaluated. The intention was to establish the minimum Observation Band length required to produce satisfactory results.

Validity of the Exponential Growth Profile

Since the TPBM assumes an exponential growth profile for the plant additions under all circumstances, it is quite likely that the model will fail to perform as expected, especially if the actual growth profile is other than the exponential type (like linear, no growth etc.). Hence the TPBM was tested with simulated data sets having different growth profiles (of the plant additions) and the performance of the model was studied.

Adequacy of the Conformance Index

Due to the large number of possible combinations of the input parameters, the number of tests conducted by the TPBM is also equally large. As such, the entire success of the model is dependent on the sensitivity of the Conformance Index to the variations in the parameters and the accuracy of the characterestics selected by the index. Hence, the performance of the index was studied for various input variables.

3.3.1 Procedure adopted for the Investigation

In order to reduce the distortion likely to be caused by parameters other than the one being tested, their values were held fixed at some prespecified values while that of the parameter under study was being varied. The input data sets (the plant additions, plant retirements and the plant balances) necessary for the tests were generated using a

computer program. The generated data sets were deterministic with the plant balances conforming exactly to the specified Iowa type curve and had the specified average service life and the growth rate. The data sets generated were according to no-growth, linear growth or exponential growth profile of the plant additions. The performance of the model and hence that of the Conformance Index was measured by its ability to select the right average service life and the right Iowa type curve (identified by its standard deviation and designation number).

Investigation of the Transparent Band

This phase of the investigation involved testing of the sensitivity of the model to the Transparent Band length. The Observation Band length was fixed at 10 years. A linearly growing data set was used for the tests. The Transparent Band length was varied from 30% of the true average service life to that percentage of the average service life for which there will be 5% or less surviving. The following are the data parameters specified for the study:

Length of the Observation Band = 10 years,

Growth profile of the data set = Linear

Type curves used =L0, L3, L5, S(-0.5), S3, S6, R0.5, R2.5, R5.

Average Service Life=10 Years.

Number of Years for which data was generated =Maximum life + 10 Yrs.

The ranges of the parameters specified in the TPBM were,

Curves to be tested = All 31 Iowa curves

Average Service Lives to be tested = 7 to 13 years in increments of 1 year.

Growth Rates to be tested = 1.0 to 1.2 in increments of 0.01

Investigation of the Observation Band

The length of the Observation Band was varied from 3 to 10 years. Throughout this phase of the study, the length of the Transparent Band was held at the optimum value as found out in the previous phase of the tests. The rest of the data for these tests was the same as before.

Investigation of the Growth Profile

The effectiveness of the TPBM to select the right mortality characteristics of the data sets with different growth profiles was tested. The data sets generated for this test were of the following growth profiles (growth of the plant additions):

1. linear,
2. exponential, and
3. no-growth.

The optimum lengths of the Transparent Band and the Observation Band as determined from the previous phases of the tests were used. The rest of the data used was as

before.

3.3.2 Results of the Investigation

It was found that the validity of the results obtained were very much dependent on the length of the Transparent Band used. The minimum length of the Transparent Band required varied with the modal type and the peakedness of the curve being tested.

The minimum required length of the Observation Band was found to be low for lower order curves and high for higher order curves. It was found that caution is necessry in the use of the method if the actual data available is for only 4 or 5 years.

It was found in cases of a no-growth situation that the results produced are not satisfactory. It was also found that the method produces satisfactory results in case of linear growth and quite likely in exponential growth situation also.

From the test results, it was concluded that the CI might not be a good indicator of the best fitting mortality characteristics because of its frequent unreliable and inconsistent behavior.

Another finding of the study was that the right average service life will be selected for a wide range of input variables. As such, the average service life selected is insensitive to the input variables over a wide range. However, the curve type selected is quite sensitive to the

input variable specified.

The following chapter deals with how the existing TPBM was modified in the light of the results of the investigation conducted by Tharumarajah.

4. MODIFIED TRANSPARENT PLANT BALANCE METHOD

The previous chapter discussed the existing Transparent Plant Balance Method and its performance as determined by an evaluation study conducted by Tharumarajah [11]. From this study, a few of the limitations of the TPBM came to light. In this chapter, the method employed to suitably modify the TPBM and the rationale behind it will be discussed. The objective of this modification was to overcome some of the limitations and to rectify some of the inherent flaws in the process itself. The modified version of the TPBM so developed will be hereafter called the 'MTPBM'.

Before it is attempted to modify the TPBM, it is imperative to recognize the implicit assumptions incorporated in the TPBM; and understand their likely influence on the performance of the model. This would help to determine the validity of the existing model. Also, it is equally important to understand and logically explain the possible reasons for the occasional inconsistent performance of the model. The following sections contain an evaluation of the process adopted in the TPBM, possible reasons for its somewhat unsatisfactory performance and the suggested alternate approach.

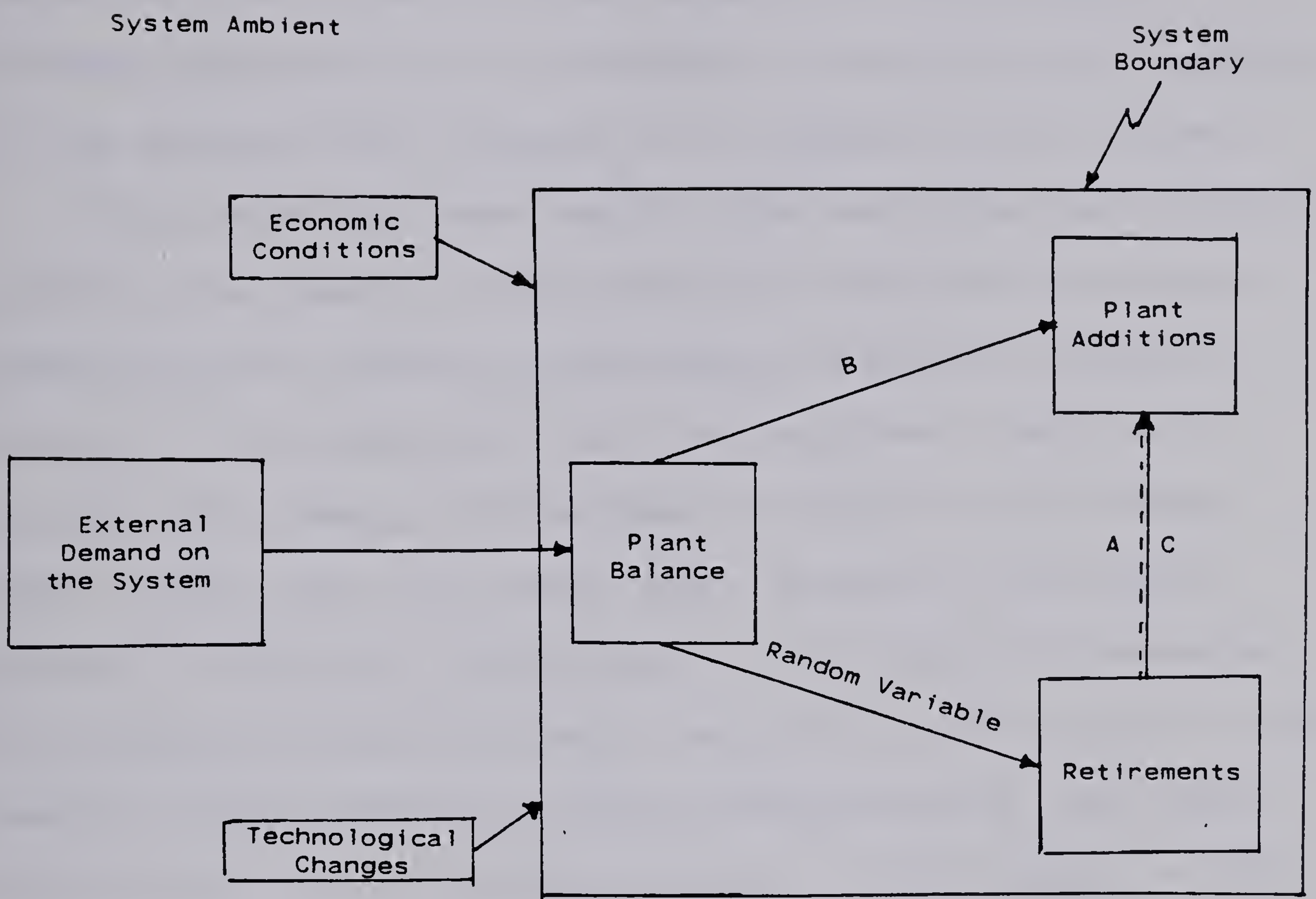
4.1 Evaluation of the TPBM

This section is an interpretive discussion of the results of the study conducted by Tharumarajah [11].

4.1.1 Process of the TPBM

The validity of the TPBM hinges on the assumption of an exponential growth profile of the plant additions. The model, as developed and being used by Edmonton Telephones, treats the plant addition as an independent variable. This allows the analyst to specify a growth profile and growth rate to the plant additions. Due to this treatment, the plant balances respond to the specified growth profile of the plant additions, thereby behaving as a dependent variable of the plant additions. In the study conducted by Tharumarjah to validate the model, the data sets required for the test were simulated not only deterministically but also with a growth profile specified to the plant additions. Consequently, the study has only confirmed that, if the plant additions grow exponentially or linearly in a real life situation, the TPBM is likely to provide satisfactory results. Hence, the questions to be answered now are 'Is it acceptable to treat the plant addition as an independent variable and specify a growth profile to it?', 'Does the plant balance really behave as a dependent variable of the plant additions in real life?', and 'What would be the performance of the model if tested with stochastic data sets (as is true in real life) rather than deterministic data sets?'.

To answer these questions, a systems configuration of an organization (in context to the process under consideration) was developed as shown in Figure 4.1. In this



Where,

A = required plant additions to compensate for any random errors in the plant balance,

B = required plant additions to meet the expected growth rate, and

C = required plant additions to compensate for the units expected to retire in the current year.

Figure 4.1 Systems Configuration of Industrial Property Response to External Demand

configuration, the system boundary is drawn around the organization and encloses the annual plant balances, the annual retirements and the annual plant additions. The demand on the system is shown external to the system boundary because we are interested in analyzing the response of the system to any changes in the demand on the system.

The system as conceived for this analysis functions as follows: the demand on the system for the service/product produced by the system is generated outside the system boundary. Any additional plant and equipment required to satisfy this demand will be made availabe by the system. Thus, at any time, the total plant balance in any plant account is directly proportional to the external demand on the system. The plant balance responds to any changes in the demand; if the demand is growing exponentially, the plant balance will also grow exponentially; if the growth of the demand is linear, the plant balance grows linearly, and so on. Though the plant balance is apparently a dependent variable of the total demand on the system, it can be considered as an independent variable with reference to the system itself. This is because, the management of any organization (system) has the option to manipulate the plant balance to meet the entire demand or a part of it.

Usually the managerial policy of any system will be to keep the plant balances growing at a specific growth rate. The plant addition only responds to the required growth rate of the plant balance. When the plant balance is required to

respond to any growth in the demand on the system, it is accomplished through the plant additions, thereby making it a dependent variable. Had the plant addition been a direct function of the required change in the plant balance alone, it would not have mattered whether the plant addition is treated as a dependent variable or otherwise. This is because, in such a case, the plant additions would have been just equal to the required change in the plant balance. However, this is not true in a real life situation because,

$$N = A + B + C + D \qquad \text{Eqn. 4.1}$$

where,

N = plant additions in the current year

A = additions to compensate for the random errors in the plant balance caused due to the randomness of the plant retirements,

B = required change in the plant balance to meet the growth rate,

C = expected retirements in the current year from the vintages of the previous years, and

D = expected retirements from the new vintage to be installed in the current year.

The last term in equation 4.1 arises because, as per convention, all plant additions are assumed to be on July 1st (though they might be scatterd all through the year) and the plant balances are as on December 31st of the same year.

Hence any plant additions in a given year will be half a year old by the end of the year and a few of the units might retire in that half a year period.

Consequently, if it is necessry to have 'X' units at the end of the year, it is essential to install 'X+F' units at the middle of the same year. Here, 'F' is the number of units expected to retire by the end of the year from the vintage installed at the middle of the same year (the value for 'F' can be derived from the Iowa survivor type curve being considered). The plant addition compensates the plant balance for the stochastic retirements and thus filters out the randomness of the plant balance to a great extent. However, the plant addition itself becomes a stochastic variable because it captures a major portion of the inherent randomness of the plant balance. Thus, the plant additions can be considerd as a stochastic variable and is governed by the corresponding retirement probabilities of all the vintages to date. It seems logical to assume that the managerial policy of any organization will usually be to maintain the plant balances at a specified growth rate to meet the changes in the demand. If this assumption is accepted, the plant additions will have to be seen as a dependent variable responding to the changes in the plant balance. Consequently, the assumption of the TPBM that the plant additions always grow exponentially does not appear to be a valid assumption. It seems more logical to assign a growth profile and growth rate to the plant balances.

Suggested Approach

In the light of the preceding discussion, it is advantageous to suitably modify the model to treat the plant balance and plant additions as independent and dependent variables respectively. This approach necessitates the extension of the plant balances into the Transparent Band instead of extending the plant additions.

In the TPBM, the only growth profile being used is the exponential growth profile. But it does not seem reasonable to assume that any property account will always grow in an exponential manner. It is not uncommon to come across stationary accounts (ie. no growth in the plant balances) or linearly growing accounts. Hence the model should have provision to specify growth profiles other than the exponential growth profile.

4.1.2 Conformance Index

It was mentioned in the previous chapter that due to the method employed for the data generation, the TPBM generates and tests a large number of data sets which are graded for the closeness of fit using the Conformance Index (CI). If more growth profiles are to be added, the number of data sets tested will increase. Under the circumstances, it is imperative to use an index which is sensitive enough to differentiate the data set with the actual growth profile and the mortality characteristics. However, the index should be reasonably insensitive to the inherent stochastic

variations in the data sets. The performance evaluation of the Conformance Index conducted by Tharumarajah indicates that the Conformance Index appears to be an inadequate index of the closeness of fit. Hence, it is necessary to take a closer look at the CI and try to understand the possible reasons behind its dubious performance. This is a very important and essential step towards the selection of an index with a favorable performance. Equation 3.3 is the mathematical equation for the Conformance Index which reduces to that shown in Equation 4.2.

$$CI = \frac{\sum_{j=1}^{n} P_j}{n^{1/2}[\sum_{j=1}^{n}(P_j - P'_j)^2]^{1/2}} \qquad \text{Eqn. 4.2}$$

where,

n = number of years in the Observation Band,

P_j = actual plant balance in the jth year of the Observation Band,

P'_j = simulated plant balance in the jth year of the Observation Band, $= \sum_{i=1}^{j}(N_i)x(S_{ij})$

N_i = plant additions in the ith year of the account,

S_{ij} = survival rate in the jth year of the property installed in the ith year of the account.

It is evident from Equation 4.2 that the CI is a function of the length of the Observation Band; it is inversely related to the root of the length of the Observation Band. Hence it does not seem reasonable to assign any predefined ranges (as is being done) to the index

as a basis for grading the closeness of fit of the data sets.

Equation 4.2 further reduces to Equation 4.3.

$$CI = R/n^{1/2} \qquad \text{Eqn. 4.3}$$

where R is the same as Equation. 4.2 except for $n^{1/2}$ in the denominator of that equation.

Now, consider two data sets 'A' and 'B' with, say, 5 and 8 data points respectively (ie. plant additions, plant retirements and plant balances for 5 and 8 years respectively). While testing these data sets, if it so happens that the ratio 'R' (Equation 4.3) in both cases work out to be equal, then the CI will have a higher value for the data set 'A' because the value of the denominator (Equation 4.3) will be $\sqrt{5}$ for data set 'A' as against $\sqrt{8}$ for data set 'B'. This is, in net effect, the same as saying that if the number of years for which the actual data is available is smaller, the selected mortality characteristics are better. It can be easily seen that the converse is true. That is, the larger the number of years for which the actual data is available, the better will be the selected mortality characteristics. In this context, it becomes necessary to differentiate between a 'good quality' fit (a fit which produces the least root mean squared error) and the best fitting mortality characteristics (mortality characteristics which accurately represent those of the data set).

Let us assume that the actual data (plant additions, plant retirements and plant balances) are available for only one year. Then it is possible to select a large number of combinations of the mortality characteristics, growth profile and growth rates which can simulate the actual data with a great deal of accuracy. Thus, even though the quality of the fit between the simulated and the actual data might be excellent, the quality of the mortality characteristics selected could be very poor. However, if the actual data is available for two years, many of the previously selected combinations of the mortality characteristics could be easily eliminated. Thus the quality of the selected mortality characteristics would be better than in the first case. However, because there are two years of actual data (which are stochastic in nature) to be simulated, the minimum possible value of the root mean squared error from all the different combinations of the mortality characteristics is quite likely to be higher than the first case where only one year of data was available. Similarly, as the number of years for which the actual data is available increases, the quality of the finally selected mortality characteristics increases. This is because it becomes increasingly difficult for the incorrect combinations of the characteristics to simulate the actual data set without producing a large error. In light of this discussion, the fact that the CI tends to have a higher value for smaller Observation Band length appears illogical.

The CI varies from zero to infinity. The arbitrary scale specified as a basis for grading the curves considers a curve as excellent if the value of the CI is 75 or higher (all the way to infinity). But since the index has a finite lower limit (zero) and an infinite upper limit, the performance of the index is likely to be biased towards the higher limit. Such a bias, if any, could be overcome if the index used has finite lower and upper limits.

The discussion under this section indicates the need for an index which is not a function of the variables of the model. Such an index, if used, could be assigned some definite and fixed ranges of values as a basis for grading the quality of the curves selected. Also, the index should preferably have finite upper and lower boundaries.

4.1.3 Data Base

For the study conducted by Tharumarajah [11], a computer program was used to generate the deterministic data sets. The process of the generation was as in Equation 4.4.

$$P_j = \sum_{i}^{j} N_i S_{ij} \qquad \text{Eqn. 4.4}$$

where,

S_{ij} = survival rate in the jth year of the account for the property installed in the ith year of the account.

N_i = plant additions in the ith year of the account.

The data so generated was tested using the TPBM. However, even the TPBM simulates the plant balances using the same process. It is likely that the ranges of the different variables specified for the TPBM are such that, during the test run, the data combination attained is the same as the combination used to generate the data set being tested. Under these conditions, the plant balances simulated by the TPBM will be identical to the actual plant balances used in the input data set because the input data set is deterministic. As a result, the CI will assume a very high value. However, in a real life situation, it is very improbable to encounter such deterministic plant accounts. Hence, the model should be tested with stochastic data sets generated using a Monte Carlo simulator.

4.2 Modified Transparent Plant Balance Method

This section deals with the procedures used to modify the TPBM.

4.2.1 Procedure of the Modified Model

In accordance to the preceding discussion, a modified version of the TPBM has been developed called the 'Modified Transparent Plant Balance Method' (MTPBM). The MTPBM treats the plant balance as an independent variable. Hence the plant balances will be extended into the Transparent Band instead of the plant additions. Plant additions will be allowed only to respond to the required growth profile and

the growth rate of the plant balances.

Two other growth profiles have been included in addition to the exponential growth profile as used in the TPBM. These growth profiles are illustrated in Figures 4.2 through 4.4. Figure 4.2 illustrates a stationary account while Figure 4.3 shows an account which is growing linearly. Figure 4.4 is the exponential growth profile as used in the TPBM.

The process of extending the plant balances into the Transparent Band is somewhat similar to the process used in the TPBM to extend the plant additions into the Transparent Band. The actual plant balance in the first year of the Transparent Band (shown as circles in Figures 4.2 through 4.4) is the starting value. The plant balances for each year of the Transparent Band is obtained by equations 4.5 to 4.7 for the no-growth, linear and exponential growth profiles respectively.

No-Growth: $P_{k-i} = P_{k+1}$, $i = 0$ to $k-1$. Eqn. 4.5

Linear: $P_{k-i} = P_{k+1} - (S)x(i+1)$, $i = 0$ to $k-1$. Eqn. 4.6

where S = the slope of the straight line ($s>0$).

Exponential: $P_{k-i} = P_{k-i+1}/R$, $i = 0$ to $k-1$. Eqn. 4.7

where, R = the exponent ($R\geq 1$).

Once the plant balances are generated for the Transparent Band as discussed above, the plant retirements and additions are simulated for the whole age (Transparent

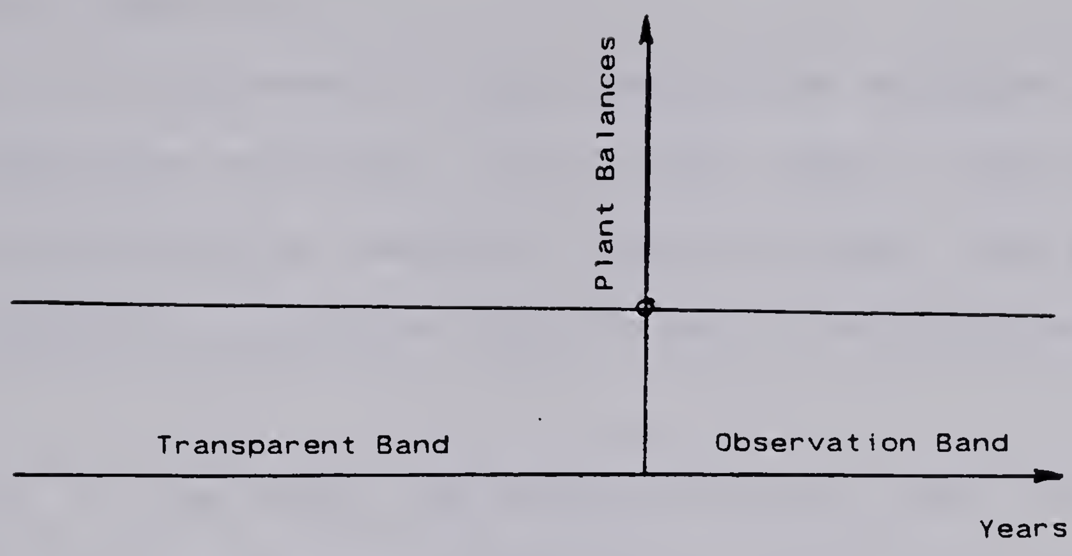


Figure 4.2 Stationary Profile of the Plant Balances for the MTPBM

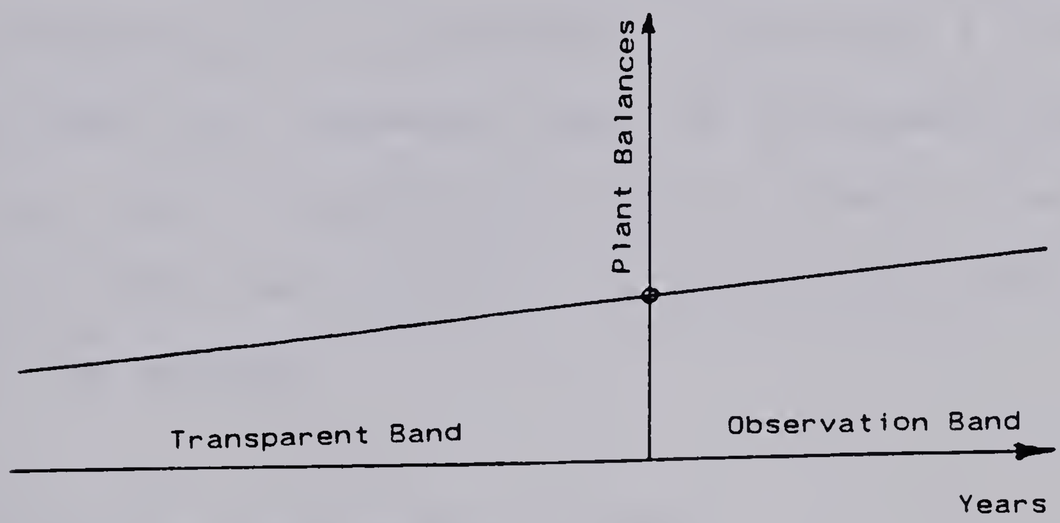


Figure 4.3 Linear Growth Profile of the MTPBM

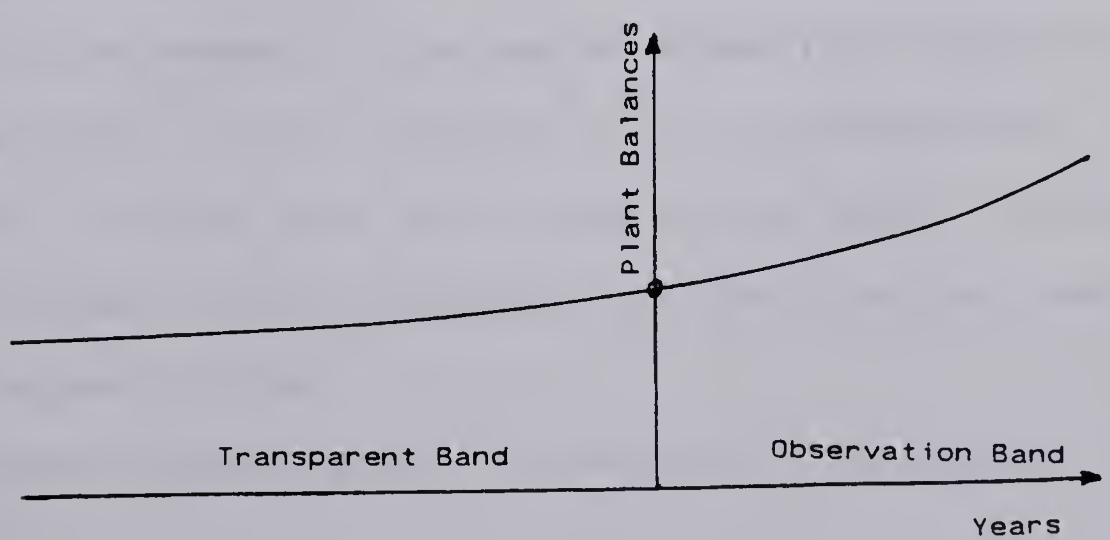


Figure 4.4 Exponential Growth Profile of the MTPBM

Band length + Observation Band length) of the account according to equation 4.1.

Due to this process of data generation employed, an initial selection criterion, as the one used in the TPBM, will not economize the computer resource usage. Hence an initial selection procedure has not been incorporated in the MTPBM.

Unlike in the TPBM, the simulated plant additions are matched to the actual plant additions of the Observation Band in the MTPBM (in the TPBM, plant balances are matched). A likely advantage of this method of matching is that, since the plant additions represent both the retirements and the growth rate, such a matching process is likely to be more sensitive to both the mortality characteristics and the growth of the account.

4.2.2 Indices for the Goodness of Fit

As two more growth profiles have been added in the MTPBM, the number of trials performed during the matching process is increased. This has enhanced the need for a reliable index of the goodness of fit. Consequently, three additional indices have been used in the model to find a suitable index for the purpose. The four indices used are:

1. Conformance Index,
2. Relative Percent Error (or Modified Conformance Index - MCI),
3. Theil's Index - Type 1 (UI), and

4. Theil's Index - Type 2 (UII).

The reason for using four indices is to study the performance of these indices and to determine which of the four indices is the most suitable to be used in the MTPBM.

An index is suitable to be used in the MTPBM if it has the ability to select the right characteristics when tested with favorable input parameters and has the ability to indicate any unfavorable parameters, if erroneously specified.

Of the four indices mentioned above, the Conformance Index has already been discussed in detail. However, a brief disusion of the CI follows because of a subtle difference due to the change in the matching process. The remaining three indices will be discussed in greater detail.

Conformance Index

Due to the difference in the matching process, the Conformance Index is given by equation 4.8.

$$CI = \frac{\sum_{j}^{n} N_j / n}{[\sum_{j}^{n} (N_j - A_j)^2 / n]^{1/2}} \qquad \text{Eqn. 4.8}$$

where,

n = number of years in the Observation Band,

N_j = actual plant additions in the jth year of the Observation Band,

A_j = simulated plant additions in the jth year of the Observation Band,

Equation 4.8 differs from equation 3.3 in that the plant

additions are used in place of plant balances.

Modified Conformance Index

The Modified Conformance Index represents the relative percent error (RPE) given by equation 4.9.

$$MCI = RPE = \sum_{j=1}^{n} |(N_j - A_j)| \;/\; \sum_{j=1}^{n} N_j \qquad \text{Eqn. 4.9}$$

Relative Percent Error (RPE) is a good measure of the error in the central tendency. That is, any difference between the mean values of the actual and simulated plant additions can be easily detected by the index. However, since more than just the mean values ought to be compared to determine whether two samples come from the same population, the index is unlikely to perform well. Moreover, the index again has no upper limit because the numerator can assume any value between zero and infinity.

Theil's Index - Type 1

This index has been proposed by Theil [2,12] and is defined by equation 4.10.

$$UI = \frac{[\sum_{j=1}^{n}(N_j - A_j)^2/n]^{1/2}}{[\sum_{j=1}^{n}(N_j)^2/n]^{1/2} + [\sum_{j=1}^{n}(A_j)^2/n]^{1/2}} \qquad \text{Eqn. 4.10}$$

This index is bound by finite lower and upper limits; zero and one respectively. The index will be zero for a perfect fit (at which time the numerator will be zero). The index will be unity when the match is so bad that all the A

values are either zero or the negative values of N.

Consider only the numerator of the index without the root sign:

$$\Sigma(N_j - A_j)^2/n = \Sigma[(A - \bar{A}) - (N - \bar{N}) + (\bar{A} - \bar{N})]^2/n$$
$$= S_A^2 + S_N^2 + (\bar{A} - \bar{N})^2 - 2rS_A S_N$$
$$= (\bar{A} - \bar{N})^2 + (S_A - S_N)^2 + 2(1-r)S_A S_N \qquad \text{Eqn. 4.12}$$

where,

$\bar{A}$ = mean of the simulated plant additions ,

$\bar{N}$ = mean of the actual plant additions,

S_A = standard deviation of the simulated plant additions,

S_N = standard deviation of the actual plant additions,

r = correlation coefficient, and

n = Observation Band length.

Now, consider the right hand side of the equation 4.12. We have,

1. $(\bar{A} - \bar{N})$ which measures any differences in the central tendency of the two samples, the simulated and the actual plant additions. This term will be zero only if the two sample means are identical.
2. $(S_A - S_N)$ is a measure of the error due to unequal variations of the two samples - simulated and the actual plant additions. This term vanishes only if the two samples have the same standard deviation.
3. (1-r) is a term indicative of the correlation between the two samples being compared. This term will be zero only if there is a complete covariance of the two

samples.

Thus, the right hand side of the equation 4.12 will be zero only if the two samples are identical in all respects.

The preceding discussion shows that the Theil's Index (UII) appears to be a very good index for the purpose under consideration. Although the denominator of the CI is the same as the numerator of the UI, because of its other drawbacks discussed earlier, UI appears to be a better index than CI.

Theil's Index - Type 2

This index has also been specified by Theil [2,12] and is denotd by UII. The index is given by the equation 4.13.

$$UII = \frac{[\sum_{j=1}^{n} (N_j - A_j)^2/n]^{1/2}}{[\sum_{j=1}^{n} (N_j)^2/n]^{1/2}} \qquad \text{Eqn. 4.13}$$

The only difference between this and the preceding index (UI) is that the second term is missing from the denominator of equation 4.13. As a result, the index has no finite upper boundary because the numerator can vary in the range of zero to infinity.

4.2.3 Final Model and the Process

This section recapitulates the previous discussion and presents a summary of the model developed. The MTPBM extends the plant balances into the Transparent Band (as per Equations 4.5 to 4.7) and simulates the plant additions as

per equation 4.1. The simulated plant additions are matched to the actual plant additions in the Observation Band. The combination of the mortality characteristics producing the best fit is selected. The indices used are CI, MCI (Relative Percent Error), UI and UII.

Stages of the MTPBM

There are three important phases of the MTPBM. They are:

1. selection of the initial parameter,
2. data generation, and
3. final selection.

Initial Parameters

The following parameters should be specified by the analyst for the tests:

1. one of the following types of growth profile (the growth profiles have been assigned the numbers 1, 2 and 3 respectively for the use in the MTPBM computer program),
    a. no growth,
    b. linear,
    c. exponential,
2. the growth rate,
    a. zero growth rate for the first growth profile,
    b. ranges of the slopes to be tested and the incremental value for the linear growth profile,

   c. range of the growth rates and the incremental value for the exponential growth profile,

3. range of the average service lives to be tested,
4. curve types to be tested (usually all 31 type Iowa curves), and
5. length of the Transparent Band.

Data Generation

The plant balances for the Transparent Band years are generated using the specified growth profile and growth rates. Using the plant balances so generated, the plant additions are simulated. Thus, a data set is generated for each possible combination of the growth rate, average service life and the curve type. This process is represented in the form of nested 'DO' loops in Figure 4.5.

Final Selection

The model selects the ten best fitting data sets for each of the four indices used. The output of the MTPBM is shown in Figure 4.6. The computer program of the MTPBM has been listed in Appendix II[1].

The next chapter deals with the Monte Carlo simulation technique used to generate the data sets for the testing and validation of the developed model.

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[1]All the variables used in the MTPBM program have been listed in Appendix I.

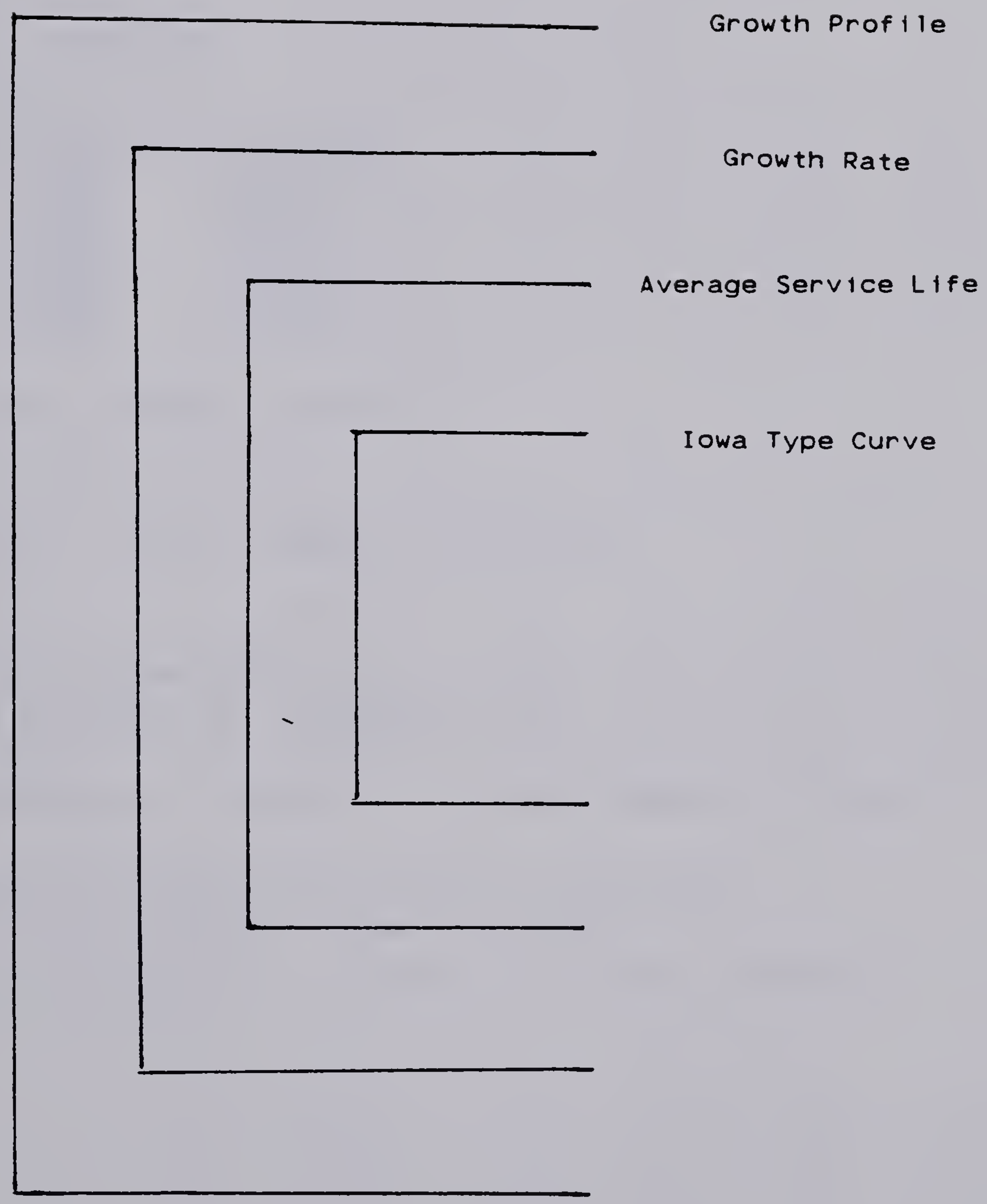

Figure 4.5 Simulation Process of the MTPBM

```
************************************

MODEL NUMBER=  29

The following straight line has been fitted TO THE OBSERVATION
BAND DATA:
SLOPE=      3010.76172
Y INTERCEPT=     125950 25000

OBSERVATION BAND DATA USED:

YEAR     PLANT IN SERVICE  GROSS ADDITIONS
1975          125920            14577
1976          128888            14868
1977          131972            15077
1978          135043            15030.
1979          138095            15157
1980          141052            15348.
1981          144022            15735
1982          146912            16162.

ACTUAL CURVE USED TO GENERATE THE DATA ABOVE
CURVE=  6
ASL=  10
GROWTH PROFILE=  2   SLOPE=      3000 0000   ACTUAL TB LENGTH=  17

THE DATA USED IN TP BO:
CURVES TESTED: ALL 31 IOWA CURVES
THE RANGE OF ASL TESTED:  7 TO  12 /INCREMENTS OF  1 YR
GROWTH: LINEAR
RANGE OF SLOPES TESTED:      2800 0000 TO       3300 0000 /INCREMENT OF       100 0000

THE FOLLOWING ARE THE VARIOUS CALCULATED VALUES FOR THE ACTUAL CURVE:
CURVE=  6   ASL=  10   OBS  BAND=  17   SLOPE=      3000 0000
CI 1=  176 47     CI 2=  0.01     MCI 1=  0 00483    MCI 2=  0.00538    REI=  0 95386   UI=  0 002835
```

Figure 4.6 Typical Output from the MTPBM Computer Program

THE FOLLOWING CURVES HAVE BEEN SELECTED BASED ON THE ORIGINAL CI
(THE GROWTH PROFILE USED IS - 2)

CURVE#	TRS. BD. LNTH.	ASL	SLP. OF LN.	CI	MCI	UI	UII
6	17	10	3000.0000	176.47	0.0048	0.002835	0.005664
6	17	10	3100.0000	172.76	0.0049	0.002896	0.005786
6	17	10	2900.0000	163.93	0.0048	0.003051	0.006097
6	17	10	3200.0000	155.59	0.0051	0.003216	0.006424
6	17	10	2800.0000	143.83	0.0051	0.003477	0.006950
6	17	10	3300.0000	133.52	0.0056	0.003749	0.007486
30	17	11	2900.0000	80.61	0.0108	0.006188	0.012399
30	17	11	3000.0000	78.18	0.0104	0.006374	0.012786
30	17	11	2800.0000	77.55	0.0117	0.006440	0.012889
5	17	10	3000.0000	72.65	0.0124	0.006872	0.013759

THE FOLLOWING CURVES HAVE BEEN SELECTED BASED ON MODIFIED CI - MCI :
(THE GROWTH PROFILE USED IS - 2)

CURVE#	TRS. BD. LNTH.	ASL	SLP. OF LN.	CI	MCI	UI	UII
6	17	10	2900.0000	163.93	0.0048	0.003051	0.006097
6	17	10	3000.0000	176.47	0.0048	0.002835	0.005664
6	17	10	3100.0000	172.76	0.0049	0.002896	0.005786
6	17	10	3200.0000	155.59	0.0051	0.003216	0.006424
6	17	10	2800.0000	143.83	0.0051	0.003477	0.006950
6	17	10	3300.0000	133.52	0.0056	0.003749	0.007486
30	17	11	3000.0000	78.18	0.0104	0.006374	0.012786
30	17	11	2900.0000	80.61	0.0108	0.006188	0.012399
30	17	11	2800.0000	77.55	0.0117	0.006440	0.012889
30	17	11	3100.0000	72.18	0.0119	0.006896	0.013848

THE FOLLOWING CURVES HAVE BEEN SELECTED BASED ON THEIL'S FORECAST COEFFICIENT TYPE 1 (UI)
(THE GROWTHPROFILE USED IS - 2)

CURVE#	TRS. BD. LNTH	ASL	SLP. OF LN.	CI	MCI	UI	UII
6	17	10	3000.0000	176.47	0.0048	0.002835	0.005664
6	17	10	3100.0000	172.76	0.0049	0.002896	0.005786
6	17	10	2900.0000	163.93	0.0048	0.003051	0.006097
6	17	10	3200.0000	155.59	0.0051	0.003216	0.006424
6	17	10	2800.0000	143.83	0.0051	0.003477	0.006950
6	17	10	3300.0000	133.52	0.0056	0.003749	0.007486
30	17	11	2900.0000	80.61	0.0108	0.006188	0.012399
30	17	11	3000.0000	78.18	0.0104	0.006374	0.012786
30	17	11	2800.0000	77.55	0.0117	0.006440	0.012889
5	17	10	3000.0000	72.65	0.0124	0.006872	0.013759

THE FOLLOWING CURVES HAVE BEEN SELECTED BASED ON THEIL'S INDEX - TYPE 2 (UII)
(THE GROWTH PROFILE USED IS - 2)

CURVE#	TRS BD. LNTH	ASL	SLP OF LN	CI	MCI	UI	UII
6	17	10	3000.0000	176.47	0.0048	0.002835	0.005664
6	17	10	3100.0000	172.76	0.0049	0.002896	0.005786
6	17	10	2900.0000	163.93	0.0048	0.003051	0.006097
6	17	10	3200.0000	155.59	0.0051	0.003216	0.006424
6	17	10	2800.0000	143.83	0.0051	0.003477	0.006950
6	17	10	3300.0000	133.52	0.0056	0.003749	0.007486
30	17	11	2900.0000	80.61	0.0108	0.006188	0.012399
30	17	11	3000.0000	78.18	0.0104	0.006374	0.012786
30	17	11	2800.0000	77.55	0.0117	0.006440	0.012889
5	17	10	3000.0000	72.65	0.0124	0.006872	0.013759

Figure 4.6 Continued from the Previous Page.

5. MONTE CARLO SIMULATOR

Life analysis and the related calculations involve estimates that are derived through statistical analysis of the past experience. The historical records by their very nature are subject to inherent random errors due to the stochastic nature of the processes generating the data. This data might be further distorted by some unusual or unnatural happenings extraneous to the process generating the data. Hence such data will prove particularly troublesome if used during the developmental stages of any methods of life analysis. Hence, during the developmental stages of any new method, it will be essential to have an undistorted data base with known parameters. Otherwise, it will be next to impossible to study the behavior of the model under development and its response to the changing input parameters. Yet another important factor to be considered is the difficulty of obtaining a sufficient number of real life data sets which have already been actuarially analyzed to determine their characteristics.

Computer simulation techniques are especially helpful to cater to these needs of the data base because it is possible to simulate stochastic data sets of known characteristics using simulation techniques. Such a procedure of testing the models being developed with stochastically simulated data has become a widely accepted standard in the field of life analysis. As a result, it is advantageous to simulate very closely controlled test data

sets as a means of observing, testing and developing the MTPBM. The Monte Carlo simulation technique is such a method of simulating stochastic data sets with known input parameters.

5.1 Principle of the Monte Carlo Simulation

The underlying principle of Monte Carlo simulation is the 'Law of Large Numbers' developed by James Bernoulli frim which the following theorem has been derived.

Theorem:

Let 'x' be the number of successes in 'n' independent trials with a constant probability 'p'. If 'ϵ' is an infinitesimally small positive number, the probability of the inequality in equation 5.1

$$|(x/n) - p| < \epsilon \qquad \text{Eqn. 5.1}$$

tends to unity as 'n' approaches infinity.

The simulation technique is very well suited for the study of physical property. This is because, if there are 'N' units in a given vintage, they can be viewed as 'N' independent trials with each trial having independent outcomes. The 'outcome' of a particular unit under consideration (trial) will be its retirement in year one, or year two, or year three, and so on, with the probability of retirement in each year being given by the ordinate value of the specified retirement frequency curve. According to White

[13],

> 'The observed mortality experience of a group of related property units may be viewed as a random sample from some parent population. Viewed in this manner, the objective of life analysis studies is to estimate the parameters (ie. dispersion and ASL) of the parent population from the observed sample. In order to develop a realistic model of retirement experience, then, it is only necessary to reverse the process of an ordinary life analysis study. That is, a random sample should be extracted from a parent population that is described by a known dispersion and ASL.'

5.2 Simulation Process

This section describes the basic Monte Carlo technique with reference to life analysis.

The objective of the simulation is to stochastically generate the plant retirements given the plant additions and the other mortality characteristics governing the property under study. The method involves the generation of a set of uniformly distributed random numbers in the range of zero and unity. There will be as many random numbers in the set as there are units in the vintage under consideration. Thus each random number in the set represents one specific unit in the vintage. A cumulative retirement frequency curve is calculated next from the specified retirement frequency curve. Now, the magnitude of each random number is matched to the ordinate values of the cumulative frequency distribution curve. The corresponding abscissa values determine the retirement ages of the unit being represented by that random number.

As a numerical example, let there be 1,000 units in a specific vintage. Let the specified Iowa curve type be L0 and the average service life be 9 years (L0-9 curve). A cumulative retirement frequency curve is now calculated by the cumulative addition of the ordinate values of the standard frequency distribution curve of L0-9 type. To simulate the retirement age of, say, the first unit in the vintage, a random number in the inclusive range of zero to one is drawn from an uniformly distributed population. The age of retirement of the unit under consideration is the abscissa value (of the L0-9 cumulative frequency curve) corresponding to the ordinate of the same magnitude as that of the random number being used. This procedure is repeated for all the 1,000 units, each time with a newly drawn random number, to simulate the retirement ages of all the units in that vintage. This simulated retirement distribution will be a random sample drawn from the parent population with a distribution of L0-9 type. However, the accuracy of the method is dependent on the number of trials made because of the underlying theorem that was mentioned earlier. Only if the number of trials are large enough, the simulated retirement distribution curve from all the vintages will be close to the specified distribution type curve.

The retirement frequency curve is a continuous function. As such, it will be difficult to find a point on the curve equal to a discrete value. Consequently, the cumulative frequency distribution curve is divided into

equal intervals of one year each thereby forming a discrete function. As a result, the retirements are simulated to occur during an age interval of one year duration rather than at a specific point in time.

5.3 Computer Model of the Simulator

A computer program was written for the Monte Carlo Simulator which has been listed in Appendix IV[2]. The computer model has provision for the simulation of the retirements using either the expected values or the random values. In expected value simulation, all the retirements from every vintage will conform exactly to the specified frequency distribution curve. Expected value simulation is not the 'Monte Carlo' method of simulation. The outputs from this part of the simulator were used during the debugging phase of the computer program written for the MTPBM. In the random value simulation, the resulting age-retirement frequency distribution will randomly deviate about the expected values of the specified retirement frequency curve.

As mentioned earlier, the simulation process employed is a discrete value simulation technique. Therefore, the plant additions, plant retirements and the plant balances are observed at a specific point in time. By standard convention used in life analysis studies, all plant additions are assumed to be on July 1st. All the retirements are assumed to be on December 31st of the corresponding

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[2]All the variables used in the simulator program have been listed in Appendix III.

year. Hence the plant balances are as observed on December 31st of each year after all the additions and retirements have occured.

The program provides for three types of growth profiles for the plant balances:

1. no-growth,
2. linear growth, and
3. exponential growth.

If the first growth profile is selected, the plant balance at which the account has to be stationary should be specified as an input parameter; this is in addition to the other input characteristics like the average service life, Iowa type curve and the total number of years for which the account has to be simulated etc.

If the growth profile used is linear, the slope (or the number of units by which the plant balance increases every year) should be specified in addition to the ones specified for the previous growth profile (except that instead of specifying the plant balance at which the account is to remain stationary, the plant balance for the first year of the account will be specified).

The input parameters for the exponential growth profile is the same as for the linear growth profile except that the exponent has to be specified instead of the slope. For example, if the required annual growth in the plant balance is 5%, the specified multiplication factor would be 1.05 and so on.

Since the 'Half-Year Convention' and discrete simulation technique are used, the first age interval is 0 - 0.5 year. The subsequent intervals are (0.5 - 1.5), (1.5 - 2.5), (2.5 - 3.5) years etc., and hence are one year intervals.

Figures 5.1 and 5.2 show the output from the simulator. The output in Figure 5.1 shows all the input variables (only a part of the output has been shown in Figure 5.1. The actual output contains similar listings for all the years of simulation). The input characteristics for the data illustrated here are:

Growth profile: Exponential with a growth rate of 1.03 (ie. 3% per year).

Starting Value: 75,000 units (in the first year of the account).

Curve Type[3]: No. 5 (ie. L2 curve).

average service life: 9 years.

The simulation has been performed for 25 years.

In year one (Figure 5.1), 75,019 units (or dollars) were installed on July 1st of which 17 units retired by the end of the 1st year leaving a plant balance of 75,002 units. The simulated retirements of the entire vintage are also listed. From the output, it is evident that it takes 25 years for the first vintage to retire completely.

In the second year, 2,607 units were installed. A total of 362 units retired from both the first and the second

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[3]The number codes used and the corresponding curve types have been listed in Appendix V.

....

MODEL NUMBER= 45

STOCHASTIC DATA GENERATION

THE INPUT VARIABLES OF THE CURVE ARE

THE GROWTH PROFILE USED IS EXPONENTIAL

START VALUE= 75000. TOTAL # OF YRS= 25 CURVE NUMBER= 5 ASL= 9 GROWTH RATE= 1 030

SEED NUMBER USED= 100.0000

YEAR 1 GROSS ADDITIONS= 75019 RETIREMENTS= 17 PLANT BALANCE= 75002

VINTAGE RETIREMENTS

(1)	17	(2)	361	(3)	1166.
(4)	2139.	(5)	3889	(6)	6045
(7)	7660.	(8)	8368	(9)	8116
(10)	7722	(11)	6117.	(12)	5162
(13)	4251.	(14)	3624	(15)	2985
(16)	2305	(17)	1682	(18)	1325
(19)	977.	(20)	523	(21)	336
(22)	163.	(23)	62	(24)	20.
(25)	4	(26)	0	(27)	0.

YEAR 2 GROSS ADDITIONS= 2607 RETIREMENTS= 362 PLANT BALANCE= 77247

VINTAGE RETIREMENTS:

(1)	1	(2)	11	(3)	40.
(4)	70	(5)	135	(6)	218
(7)	282.	(8)	320	(9)	271.
(10)	292.	(11)	192	(12)	191
(13)	119	(14)	120	(15)	85
(16)	82	(17)	66	(18)	39
(19)	36	(20)	14	(21)	11
(22)	9	(23)	2	(24)	1

Figure 5.1 Monte Carlo Simulator Output - Detailed Simulation Data

YEAR	PLANT ADDITIONS	PLANT RETIREMENTS	PLANT BALANCES
1	75019.	17.	75002.
2	2607.	362.	77247.
3	3444.	1177.	79514.
4	4605.	2198.	81921.
5	6488.	4028.	84381.
6	9009.	6391.	86999.
7	11002.	8329.	89672.
8	12106.	9544.	92234.
9	12647.	10012.	94869.
10	13451.	10430.	97890.
11	12691.	9897.	100684.
12	13116.	9862.	103938.
13	13342.	10368.	106912.
14	14034.	10754.	110192.
15	14511.	11327.	113376.
16	15115.	11689	116802.
17	15508.	11655.	120655.
18	15555.	12165.	124045.
19	16247.	12647.	127645.
20	16707.	12629.	131723.
21	16952.	13237.	135438.
22	17708.	13539.	139607.
23	18153.	14152.	143608.
24	18904.	14843.	147669.
25	19730.	15063.	152336.

Figure 5.2 Monte Carlo Simulator Output - Summary of Simulated Data

vintages leaving a plant balance of 77,247 units (thus the total retirements every year contain the retirements from all the preceding and the current vintages. The plant balance is also from all the vintages to date). Again, the complete retirements of the second vintage are listed. Figure 5.2 shows a summary of the plant account for the 25 years period specified. The program lists the yearly plant additions, yearly plant retirements and the yearly plant balances.

5.4 General Equation for Simulated Plant Additions

The general functioning of the simulator has been explained in the previous sections. Details of the simulation process employed to calculate the annual plant additions will be discussed in this section.

The following notation will be used during the discussion:

i = year of the study where $i=1,2,3,\ldots,n$,

n = number of years for which the property account is being simulated,

m = maximum age when all units have been retired from a given vintage,

N_i = number of units installed in the ith year of the account,

p_{ij} = probability of retirement in the jth year of the account for a unit installed in the ith year of the account,

P_1 = probability of retirement of any unit before the end of the first year of the vintage (ie. between July 1st and December 31st of the first year of the vintage),

X_{ij} = actual retirements that occur in year j from the vintage of the year i (given the probability p_{ij}),

$E(X_{ij})$ = expected value of the random variable X_{ij}.

The process will be illustrated for a no-growth account from which a general equation will be developed for all the three growth profiles.

Let 'B' be the stationary plant balance of the account (ie. the plant balance each year will assume a stochastic value about 'B').

N_1 = Plant additions in the first year of the account

$= B + E(X_{11})$

$= B + N_1P_1$

$\therefore N_1 = B/(1-P_1)$

However, the actual plant balance by the end of the year

$= N_1 - X_{11}$

$\therefore$ Plant additions for the 2nd year $= [B - (N_1 - X_{11})] + E(X_{12}) + E(X_{22})$

$\therefore N_2 = [B - (N_1 - X_{11})] + N_1p_{12} + N_2P_1$

Now,

Plant additions for year 3

$= [B - (N_1 - X_{11} - X_{12}) - (N_2 - X_{22})] + E(X_{13}) + E(X_{23}) + E(X_{33})$

$$= [B - (N_1 - X_{11} - X_{12}) - (N_2 - X_{22})] + [N_1 p_{13} + N_2 p_{23}] + N_3 P_1$$

In general, for a no-growth account,

$$N = A + C + D \qquad \text{Eqn. 5.1}$$

where,

N = plant additions for any given year

A = additions to compensate for the random errors, if any, in the plant balances

C = additions to Compensate for expected retirements in the present year from all the previous vintages

D = additions to compensate for the expected retirements before the end of the year from the new vintage to be installed in the present year

Equation 6.1 is for a stationary plant account. For an account with a linearly or exponentially growing plant balance, an additional term will be required as below:

$$N = A + B + C + D \qquad \text{Eqn. 5.2}$$

where,

B = additions to maintain the required growth, if any, of the plant balances

The first factor on the right hand side of the equation 5.2 arises because the realized values of the retirements for the previous vintages will be different from the corresponding expected values used to determine the plant

additions of the immediately preceding year. If there is a negative error, ie. more units are remaining than expected, the error will be compensated by a corresponding reduction of the plant additions, and vice versa. However, the minimum possible plant addition is zero units thereby implicitly assuming that no unit will be retired just because it is in excess of the required number of units to meet the demand.

The data sets used for the performance evaluation of the MTPBM have been generated using this simulator. The performance evaluation has been discussed in detail in the next chapter.

6. PERFORMANCE EVALUATION

The performance of any newly developed model should be thoroughly investigated and understood before the model is used for practical applications. As such, this chapter deals with the various tests conducted on the MTPBM, the results obtained from these tests and an interpretive discussion of the results so obtained. The data sets used for the purpose have been simulated using the Monte Carlo Simulator.

Before proceeding any further, a few definitions necessary to understand the discussion to follow are provided.

An actual parameter (for example, actual Observation Band length, actual Transparent Band length etc.) is the value of the parameter with which the data set being studied has been simulated in the Monte Carlo Simulator.

A specified parameter is one which has been specified by the analyst as input to the MTPBM while conducting the tests. In real practice, the actual parameters will be unknown (the model will be used to determine the actual parameters given the available data set). Therefore, it is quite likely that the specified parameters will be different from the actual parameters. The ability of the model to determine this difference between the actual and the specified parameters, if the specified parameters are erroneous (conversely, the ability of the model to identify matching specified parameters, if the specifications are accurate) is under investigation here.

It is essential to study the performance of the model from various perspectives. The entire validity of the model is primarily dependent on the validity of the assumption that a partial actual data is sufficient to differentiate any data set with a given combination of mortality characteristics from other data sets of different mortality characteristics. Such being the case, it is imperative to test the validity of this assumption. Hence the performance of the model will be evaluated for different Observation Band lengths.

Given the required Observation Band length, the performance of the model is likely to be sensitive to the Transparent Band length specified (if the actual Transparent Band length is not known) by the analyst. Therefore the performance of the model will be tested for different specified lengths of the Transparent Band.

The next parameter likely to have a significant effect on the performance of the model is the specified growth profile. This is essential because, if the analyst unkowingly specifies an incorrect growth profile, the model might behave in a different manner than expected. Hence the sensitivity of the model to the specified growth profile will be subject to investigation in the third section of this chapter.

Finally, since the performance of the model is directly correlated to the performance of the various indices used in the model, the behavior of the four indices used in the

model will be observed during the three types of tests mentioned above. This is essential to understand the behavior of these indices for varying input parameters. It is of course recognized that the general behavioral pattern of these indices will be somewhat similar. This high correlation can be expected because of the presence of the 'Root Mean Square' function in three of the four indices. However, the points of interest here are the numerical values assumed by these indices under varying conditions and the consistency of these values to fall within some range. This study will help to establish a scale of ranges for the indices to assist in the grading of the selected curves. This is essential because the ranges set for the Conformance Index has been found to be arbitrary due to the reasons already discussed elsewhere in this report.

6.1 Testing Strategy

When the model is being tested for its sensitivity to the variations of a specific input parameter, the model should not be subjected to any other disturbance. Otherwise the effects of the parameter under study on the model will be very difficult to assess. Hence, it will be important to keep the distortions likely to be caused by other parameters than the one being studied, as low as possible. With this in view, the tests on the observation band will be conducted first by specifying the actual Transparent Band length and the growth profile. This test will yield the minimum length

of the Observation Band required for a satisfactory performance of the model. The length of the Observation Band so obtained will be used in the tests to be conducted to find the sensitivity of the model to the specified Transparent Band length. This phase of the test will provide an understanding of the required Transparent Band length, given the required Observation Band length. The values for the Observation Band and the Transparent Band obtained from the first two phases of the test will be used to test the model for its sensitivity towards the specified growth profile. The behavior of the various.indices will also be under observation throughout all these tests.

6.2 Observation Band Tests

Since the available actual data is the most limiting parameter of the model, the tests on the Observation Band length have been conducted in greater detail than for the other parameters. However, the tests have been limited to the left modal, symmetrical modal and the right modal curves. The origin-modal curves have been omitted. This is because the origin-modal curves are similar to left modal curves except for the O1 curve which is similar to the symmetricl modal curves.

For each of the three types of growth profiles, two curves from each modal type have been tested; a lower order curve and a higher order curve. The results have been displayed in Figures 6.1 through 6.18. All the plottings are

against the Observation Band length expressed as a percent of the average service life. The ordinate scale has been changed to logarithmic scale in some of the plottings to accommodate all the data points. Figures 6.1 through 6.6 are for left modal curves with the actual growth profile and growth rates as specified on each figure. Figures 6.7 through 6.12 are for symmetrical modal curves with the actual parameter as shown on each figure. Figures 6.13 through 6.18 are for right modal curves for the actual parameters as shown.

The standard deviations of the Iowa type survivor curves have been used on the plots to identify the selected type curve. The dashed lines on the plots of ASL versus Observation Band and Standard Deviation versus Observation Band indicate the respective actual values used in the Monte Carlo Simulator for the simulation of the data set. For want of space, the following abbreviations have been used in all the figures throughout.

ASL (AVG. SL) - Average Service Life

CI - Original Conformance Index

MCI - Modified Conformance Index (relative % error)

OBS. BAND - Observation Band

ORIGINAL CI - Original Conformance Index

STRD. DEVIATION - Standard Deviation of the type survivor curve.

UI - Theil's Index - Type 1

UII - Theil's Index - Type 2

Left Modal Curves

The tests for the left modal curves have been conducted under two categories; lower order curves and higher order curves.

Lower Order Left Modal Curves

Figures 6.1 to 6.3 are for a lower order (L0) curve with linear, exponential and stationary growth profiles respectively.

The results of the test for the lower order, left modal curves show that the selected average service life is sensitive to the length of the Observation Band till the Observation Band length is about 20 to 30% of the average service life. When the Observation Band length is more than 30% of the average service life, the selected average service life accurately matches the actual average service life. For a linear growth account, the selected average service life appears to be quite insensitive to the Observation Band length used.

However for the correct curve type to be selected, the actual data requirement is much larger than that required for the average service life. The stationary accounts (Figure 6.3) appear to be quite sensitive to the amount of the actual data available and require an Observation Band length of 80% to 90% of the average service life.

The plots of MCI, UI, and UII will be discussed at a later time under the discussion of the behavior of the

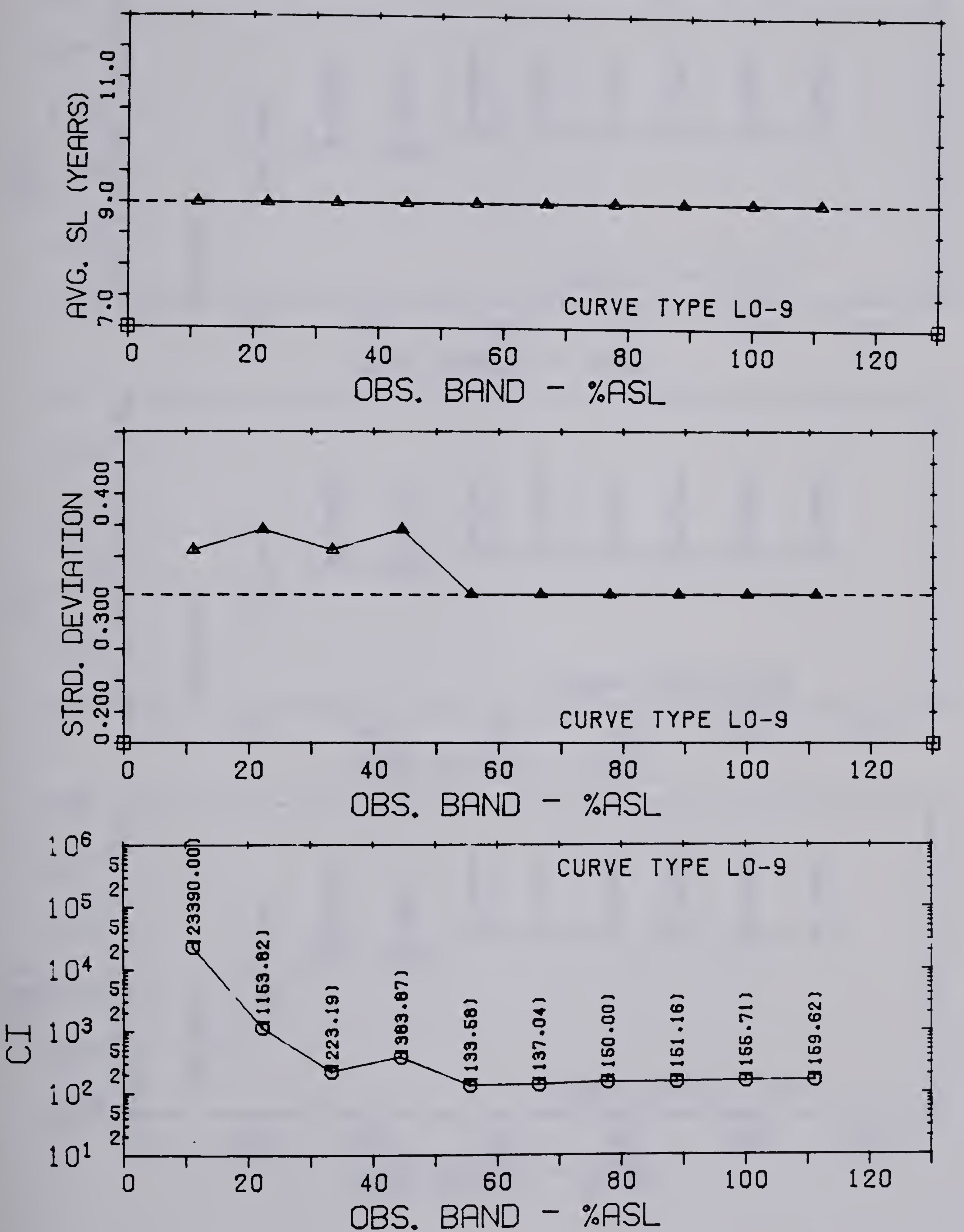
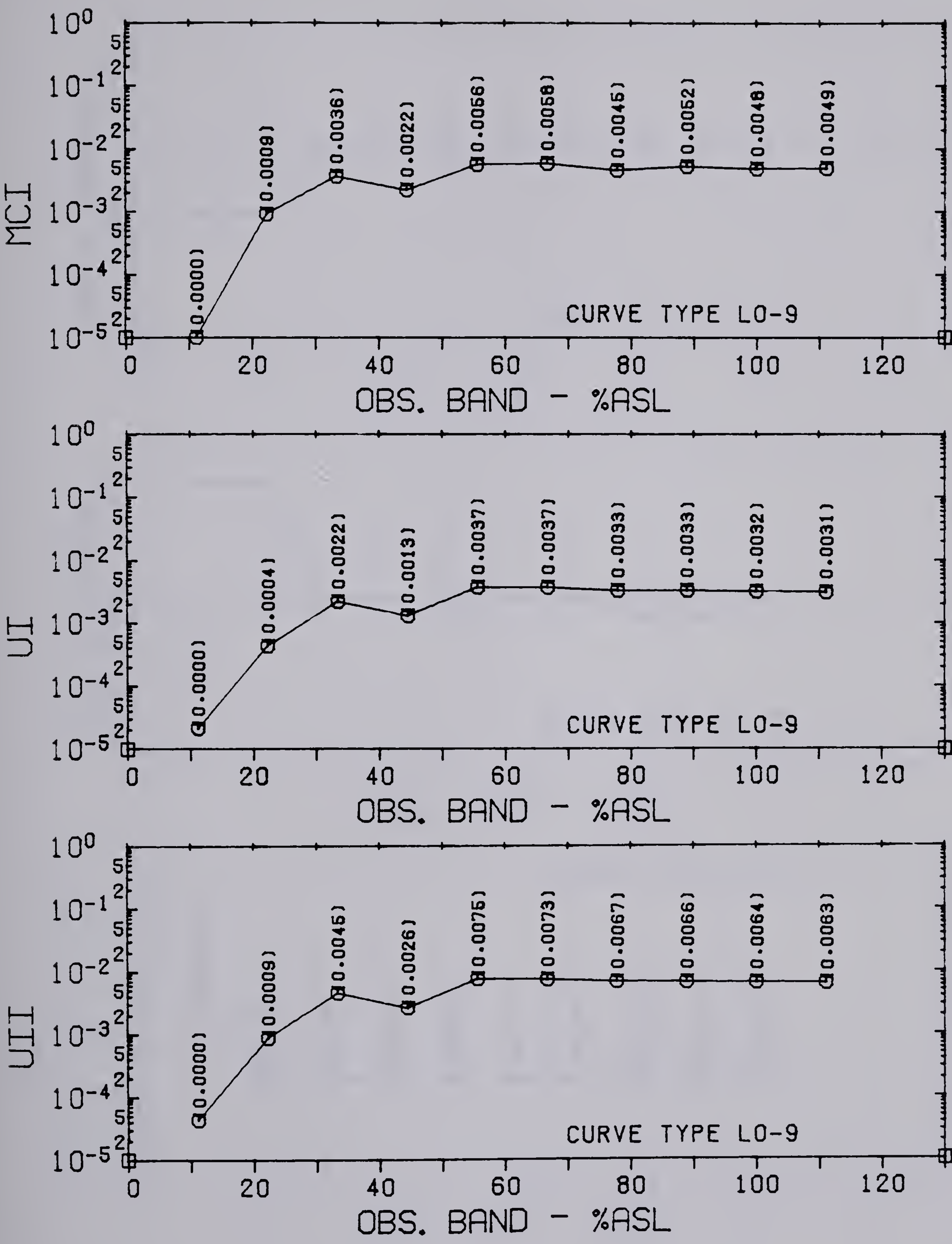


Figure 6.1 Results of the Investigation of the Observation Band Length for a L0-9 Curve With a Linear Growth Rate of 2000 Units/Yr.



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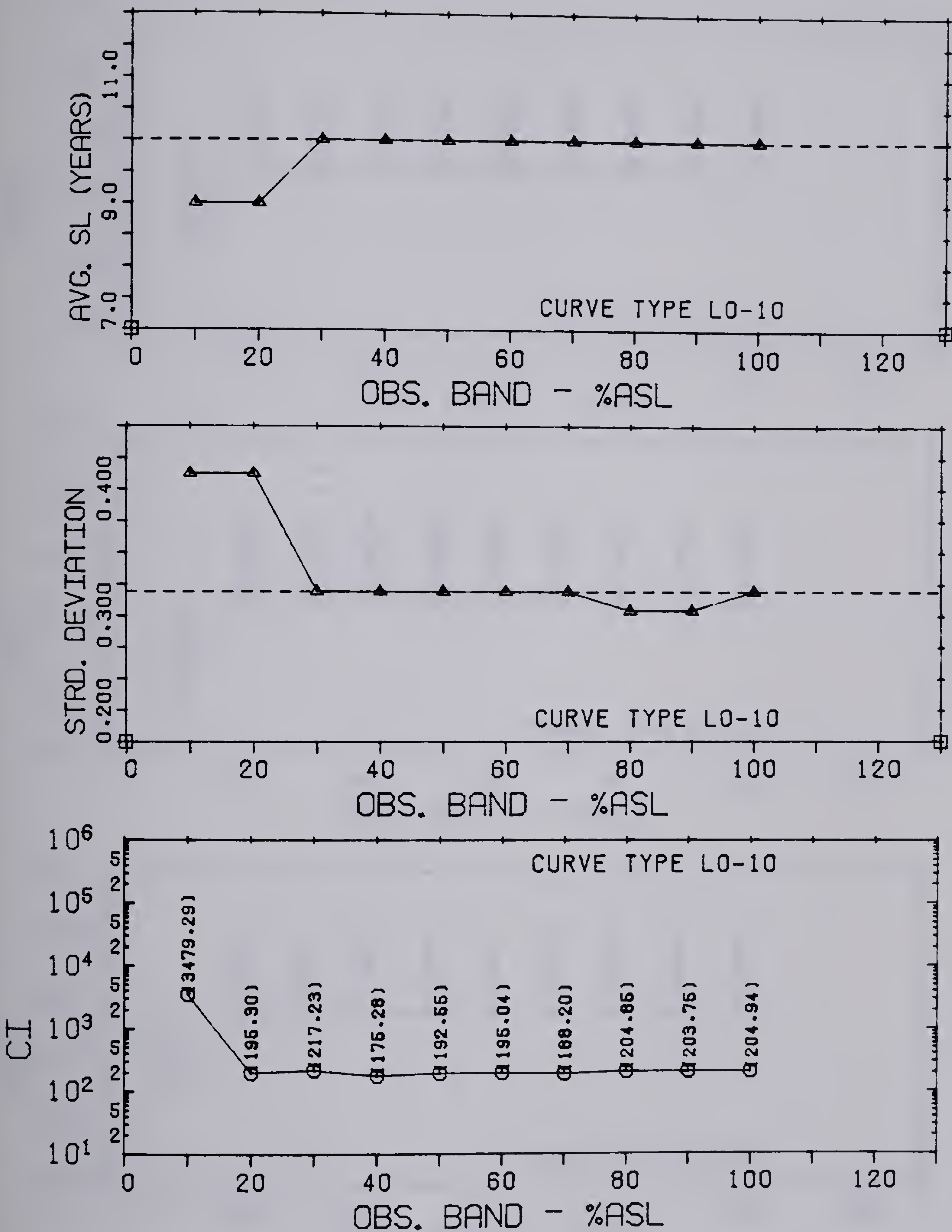
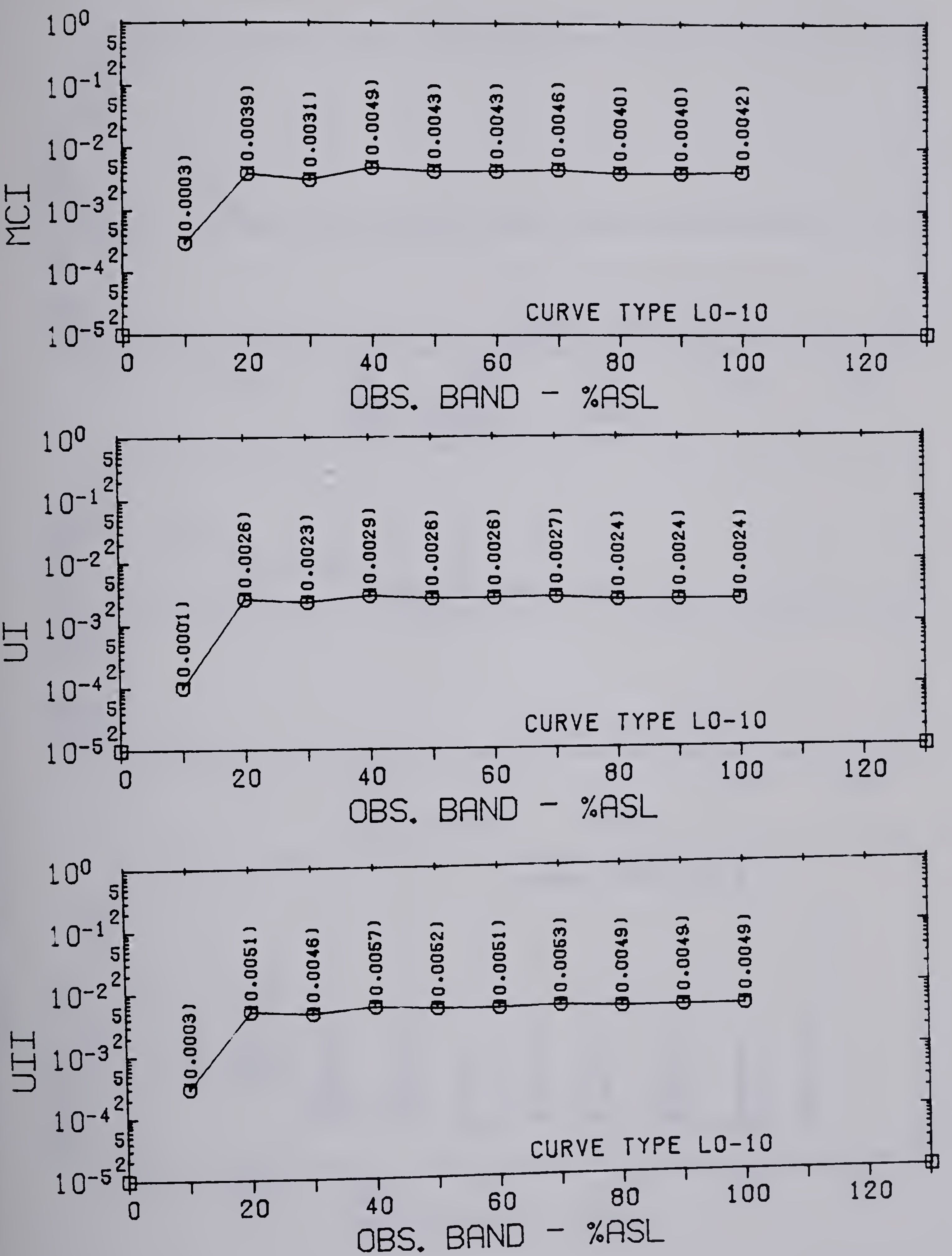


Figure 6.2 Results of the Investigation of the Observation Band Length for a L0-10 Curve With an Exponenetial Growth Rate of 1.03



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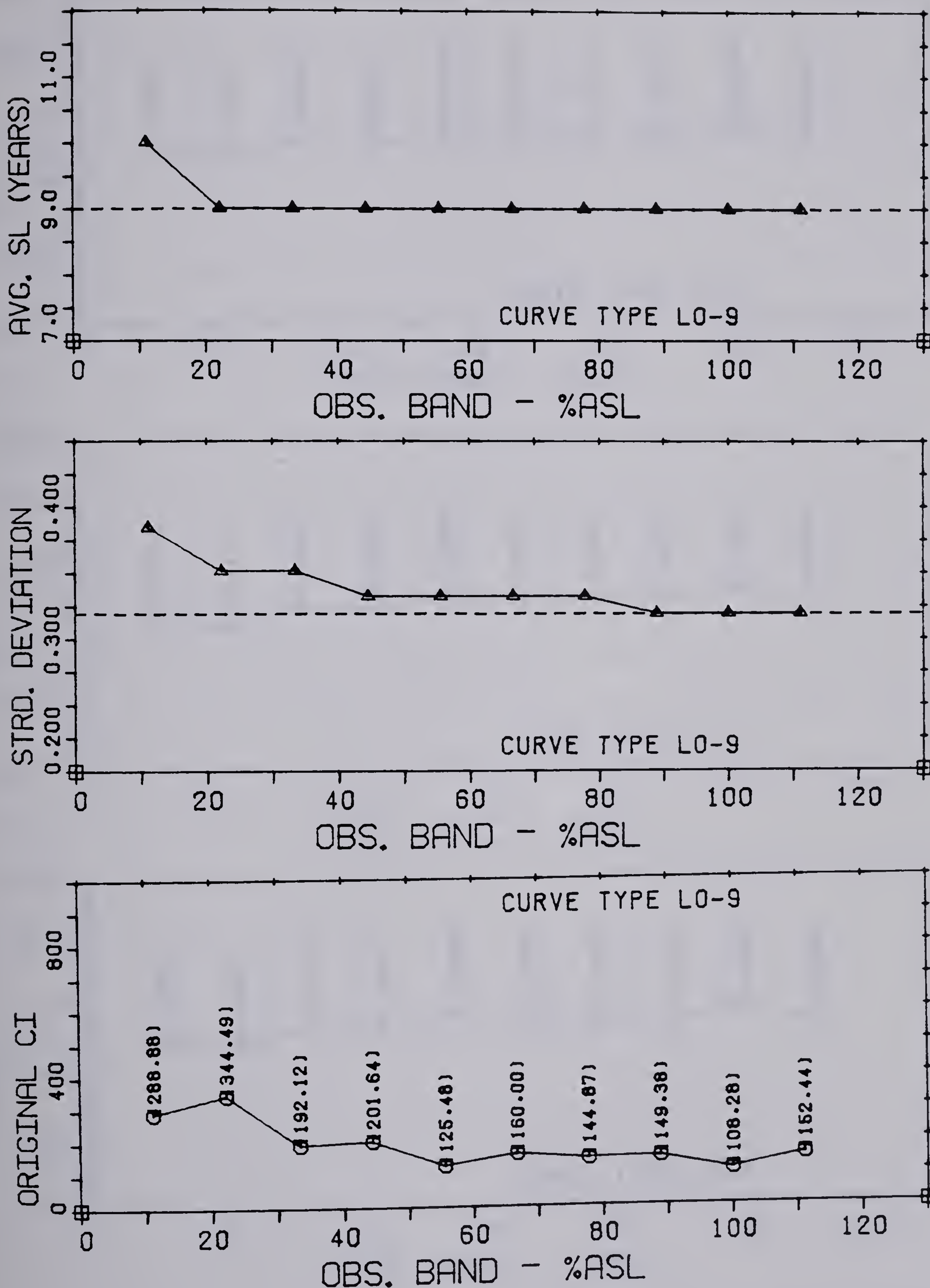


Figure 6.3 Results of the Investigation of the Observation Band Length for a L0-9 Curve With a Stationary Plant Balance

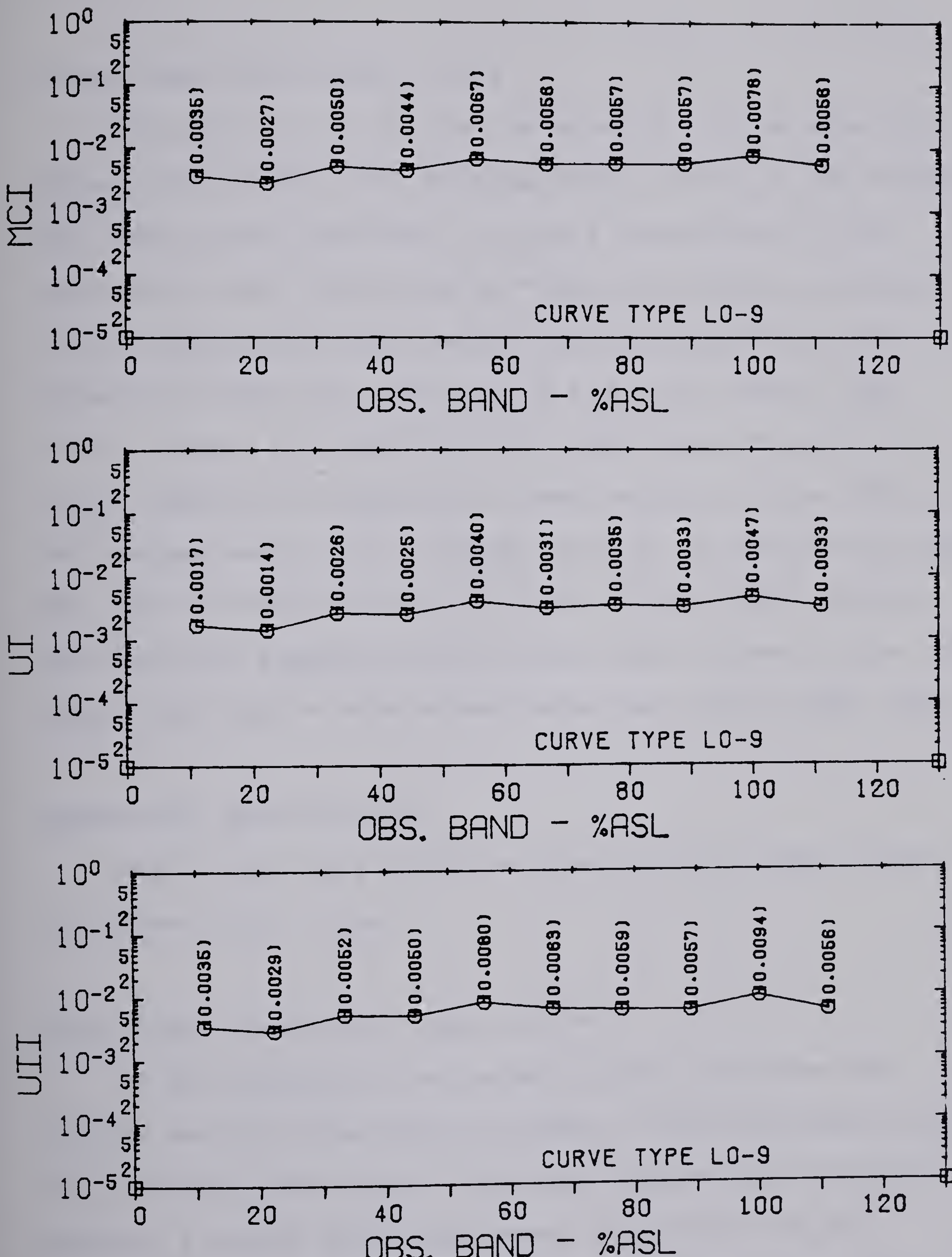


Figure 6.3 Continued from the Previous Page.

indices.

Higher Order Left Modal Curves

Figures 6.4 to 6.6 show the behavior of the model for higher order curves. The L5 type curve, which is the highest left modal curve available, is quite insensitive to the Observation Band length and provides satisfactory results with an Observation Band length as little as 10% of the average service life (Figures 6.4 and 6.6). The L4 type curve, (Figure 6.5) which is one order lower than the L5 curve, requires an Observation Band length of about 30% of the average service life. These results, in conjunction with the results obtained from the tests for the other curves discussed are suggestive that lower order curves of the left modal type require more actual data than higher order type.

Symmetrical Modal Curves

Again, the tests conducted are for lower order curves and higher order curves.

Lower Order Symmetrical Modal Curves

As in the case of left modal curves, the selected average service life does not appear to be very sensitive to the available Observation Band data (Figures 6.7 to 6.9). However, a higher data requirement is evident for the selection of the correct curve type. For both the average service life and the curve type to be accurate, the required

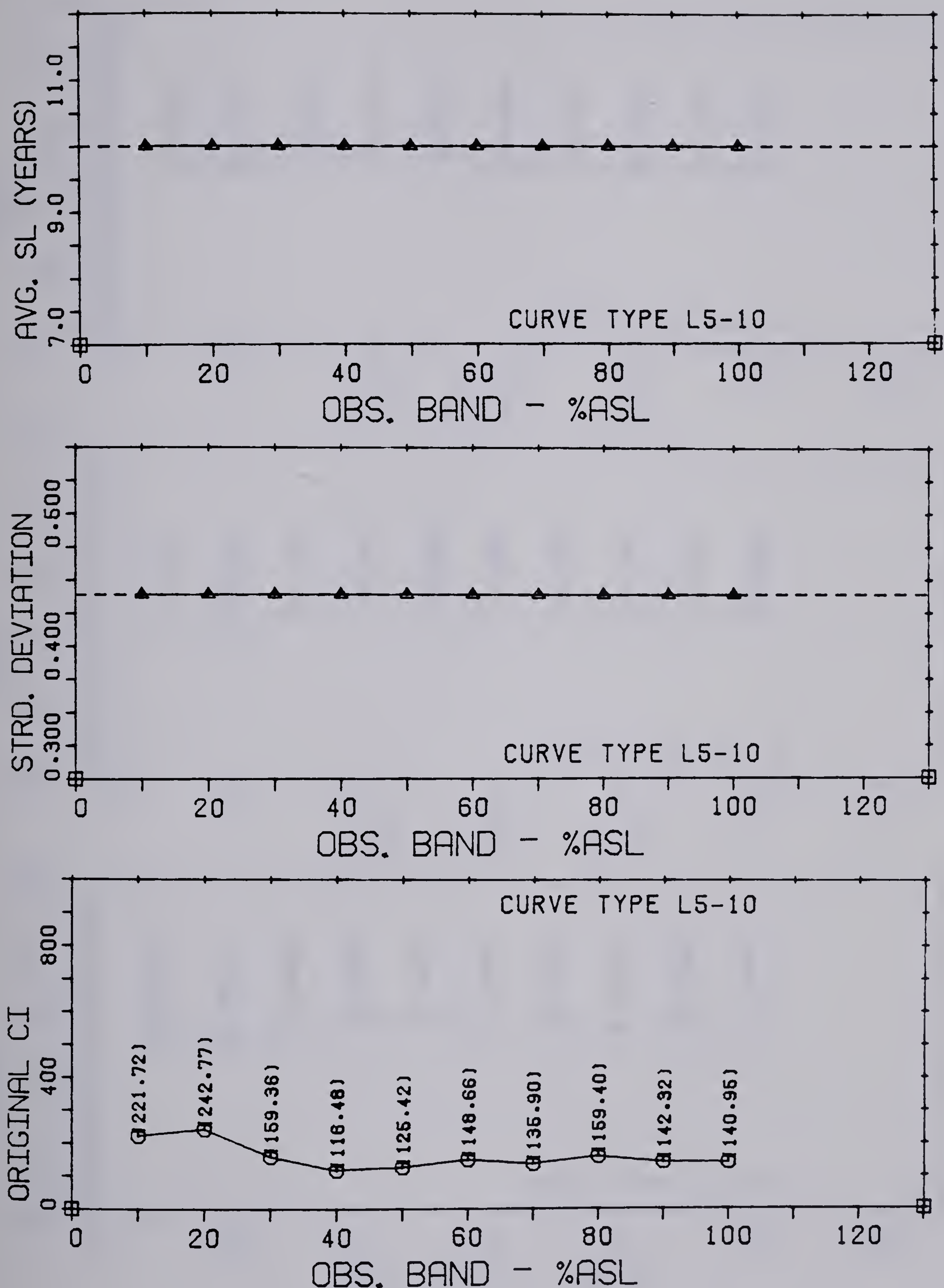
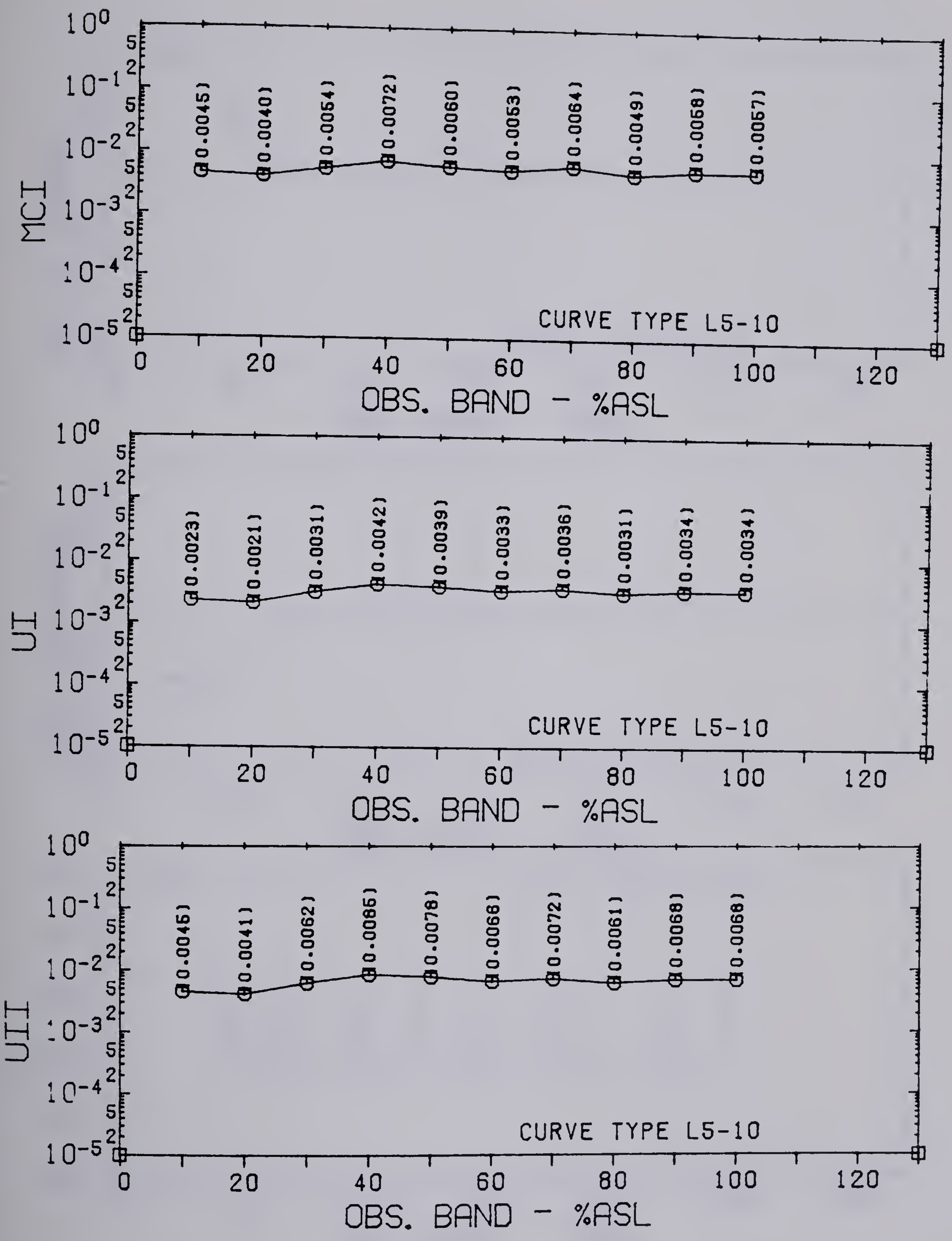


Figure 6.4 Results of the Investigation of the Observation Band Length for a L5-10 Curve With a Linear Growth Rate of 2500 Units/Yr.



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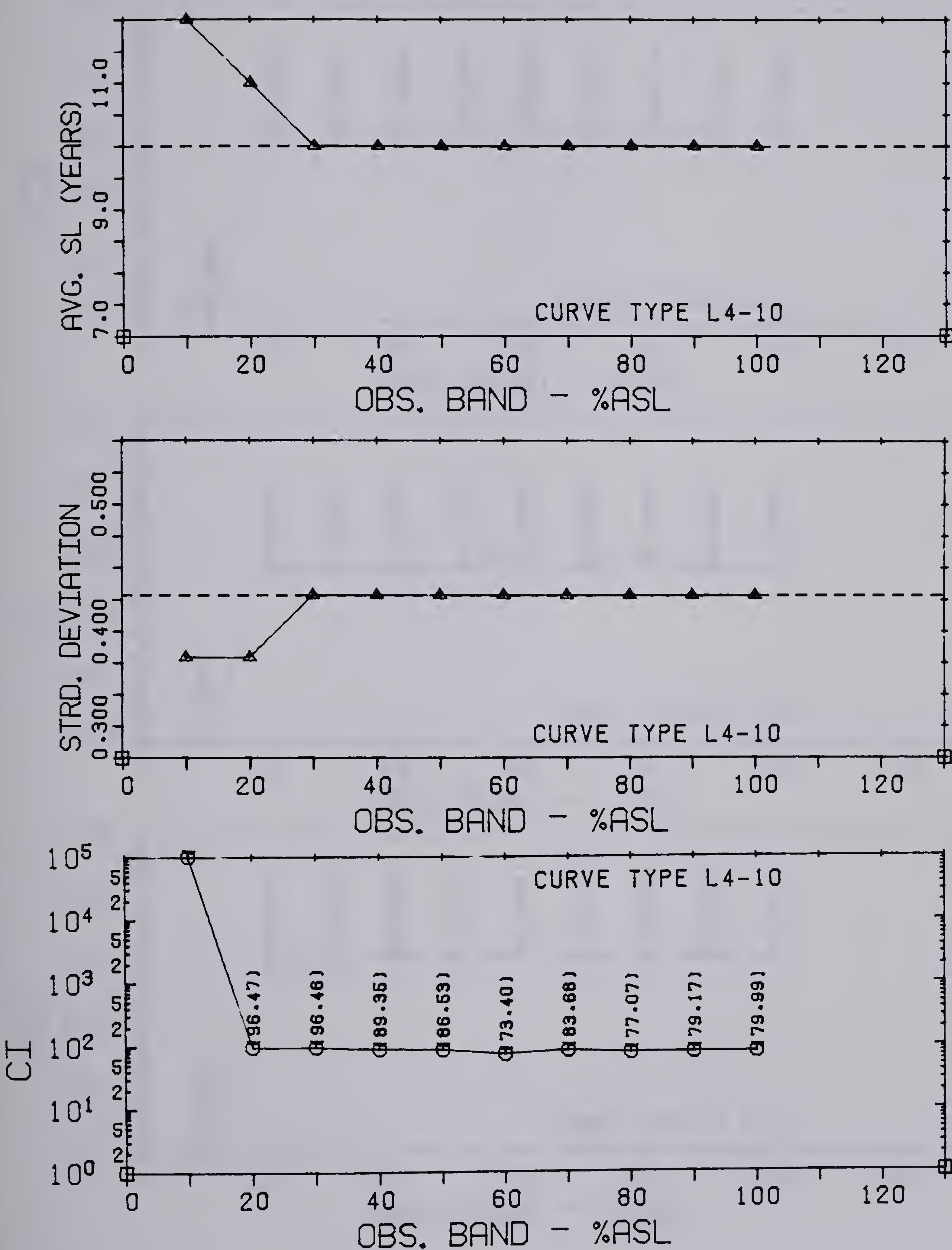
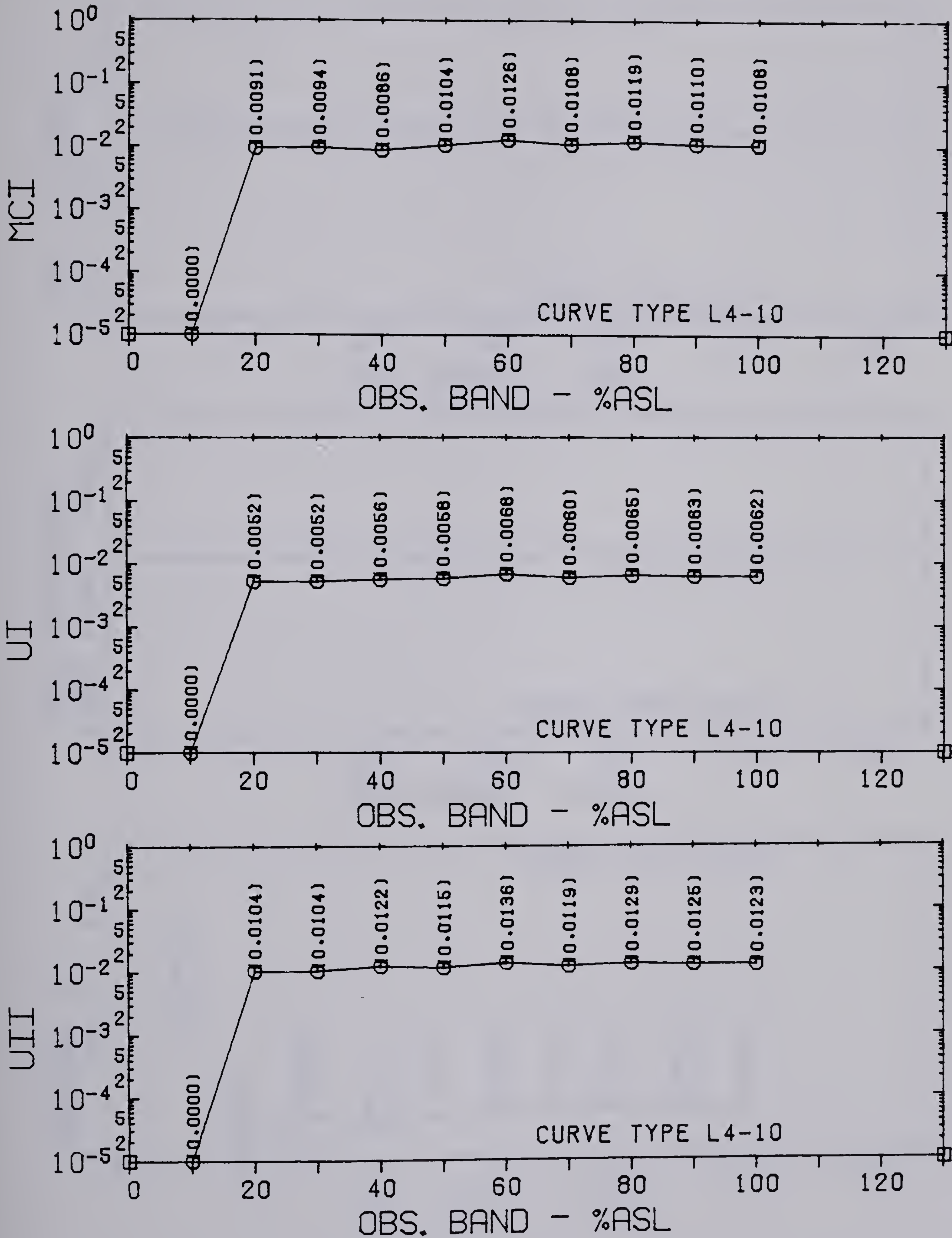


Figure 6.5 Results of the Investigation of the Observation Band Length for a L4-10 Curve With an Exponential Growth Rate of 1.02



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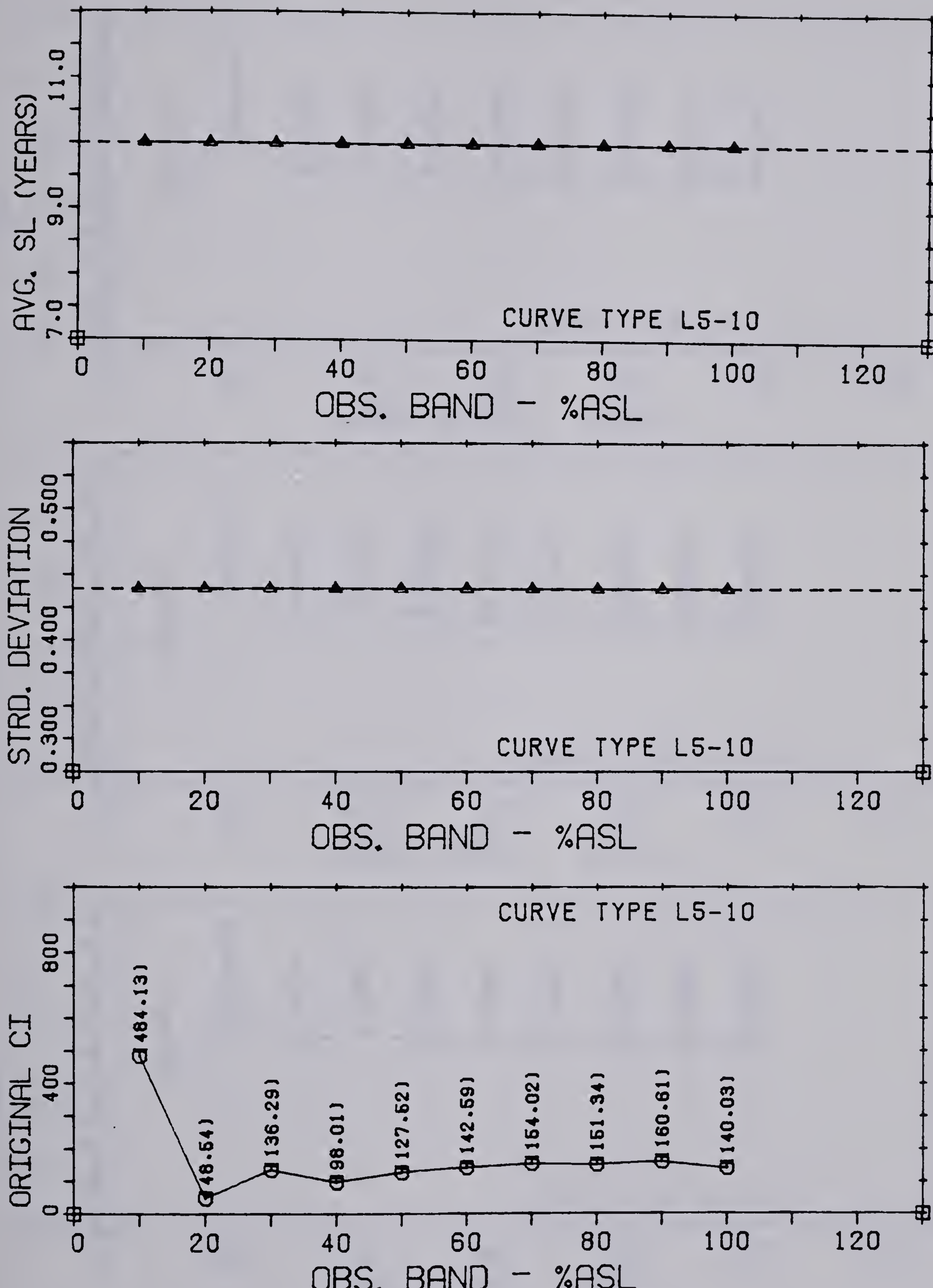
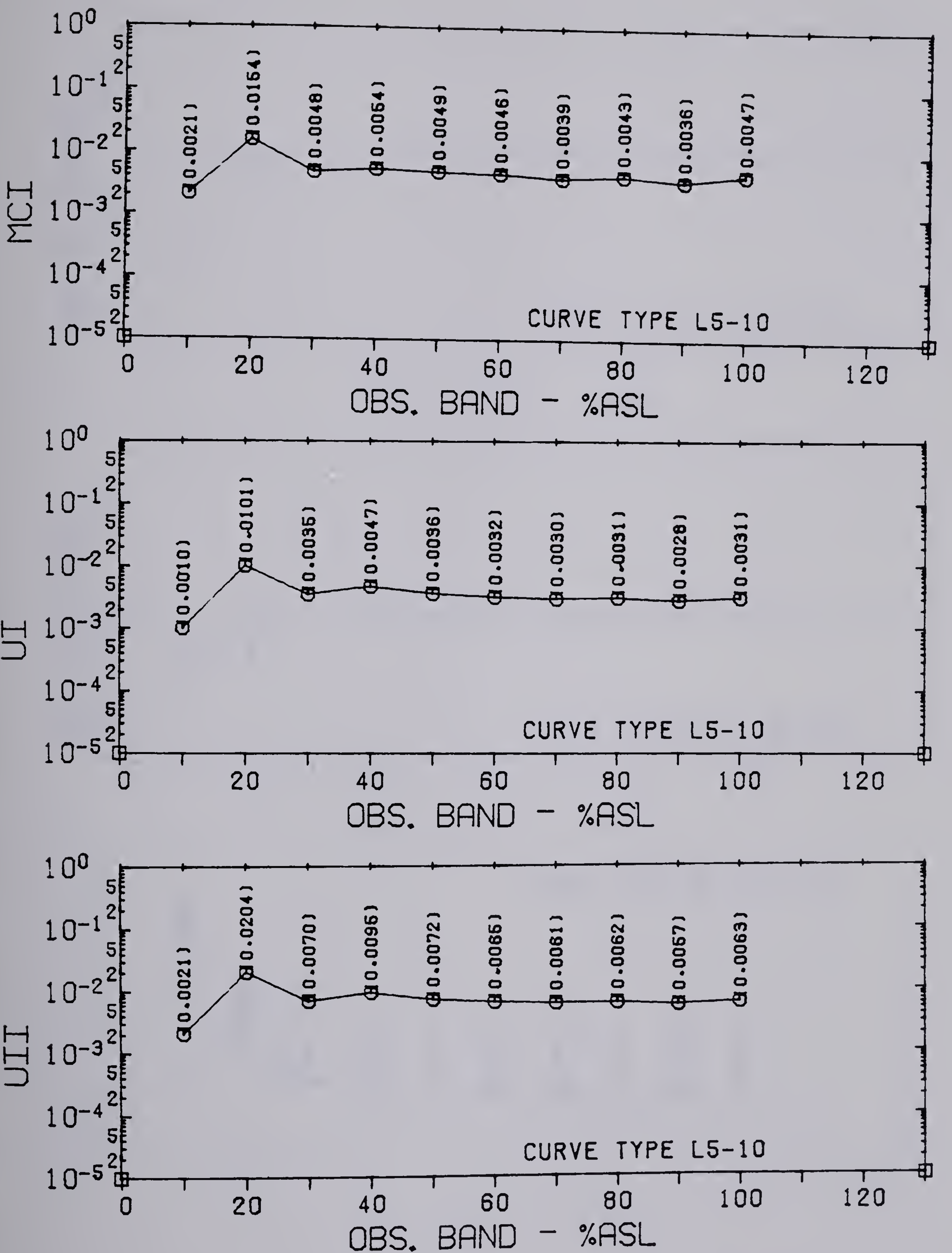


Figure 6.6 Results of the Investigation of the Observation Band Length for a L5-10 Curve With a Stationary Plant Balance



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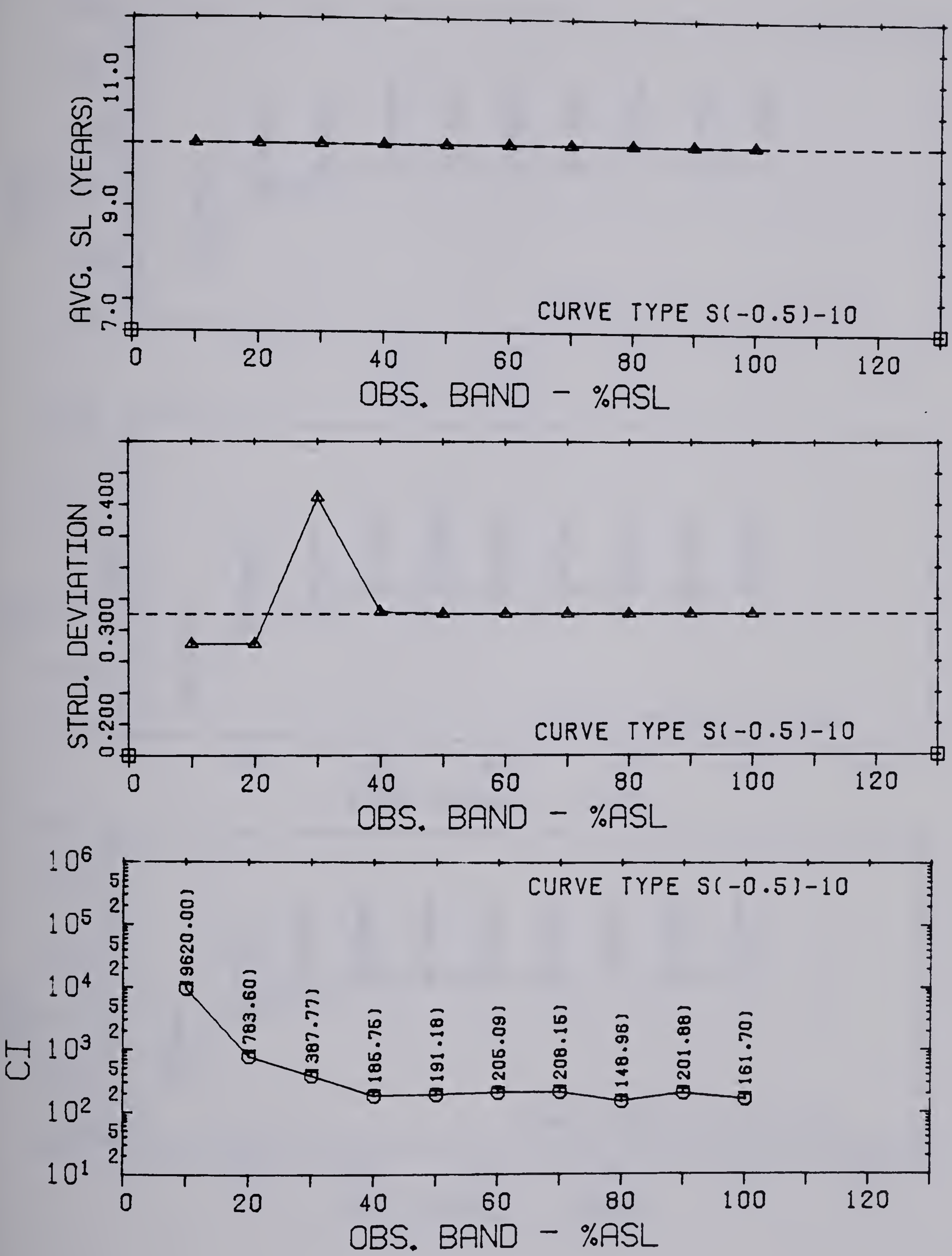
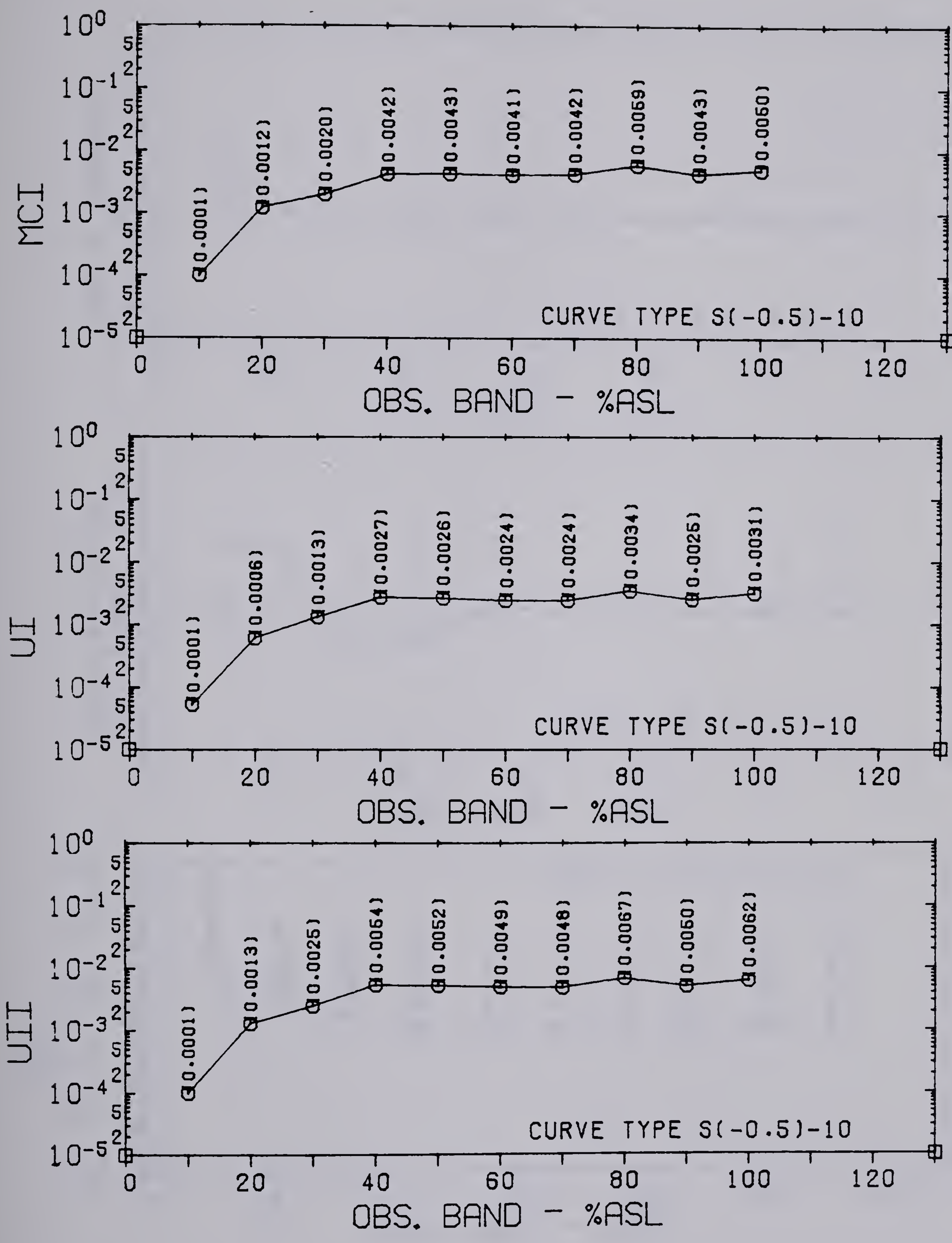


Figure 6.7 Results of the Investigation of the Observation Band Length for a S(-0.5)-10 Curve With a Linear Growth Rate of 4500 Units/Yr.



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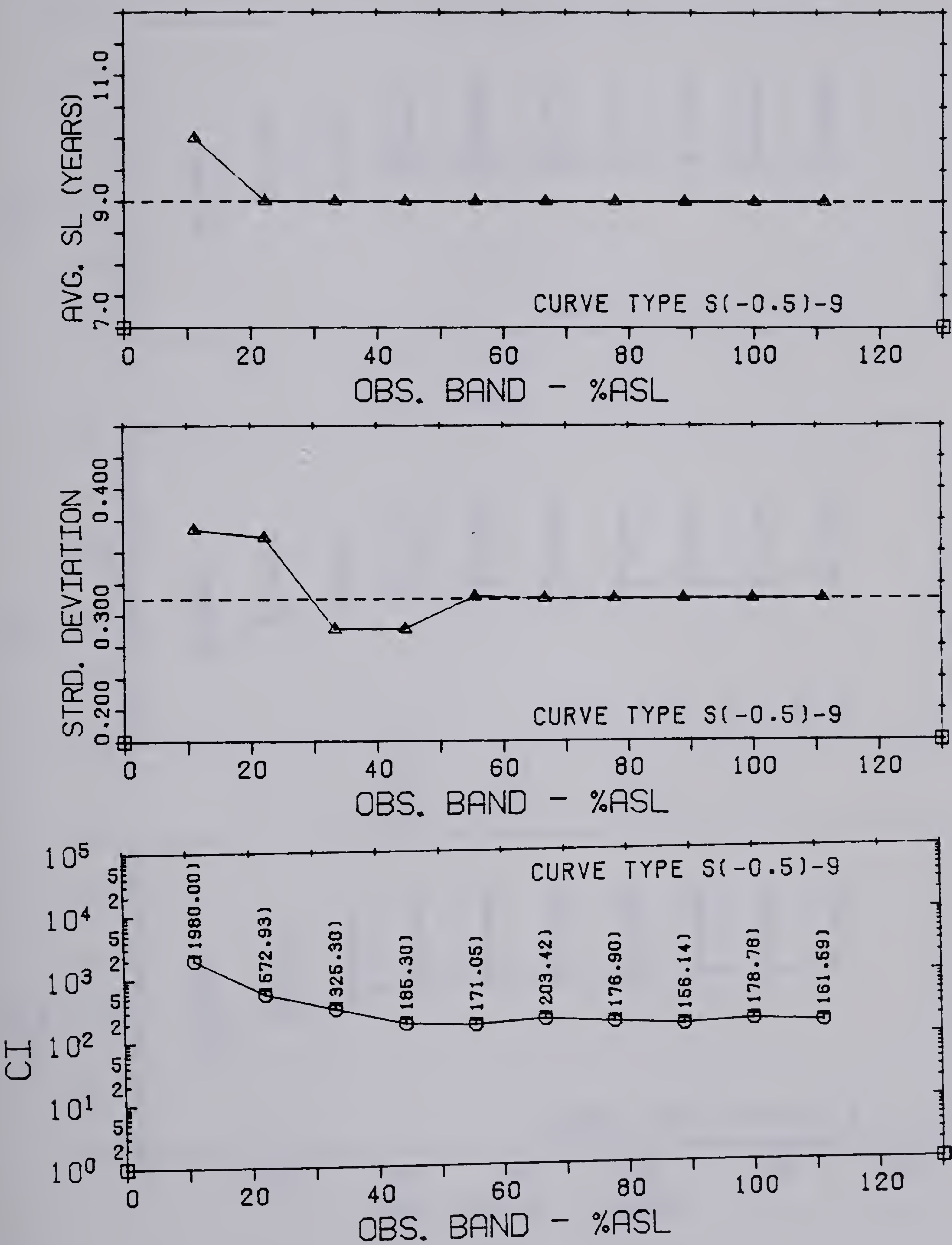
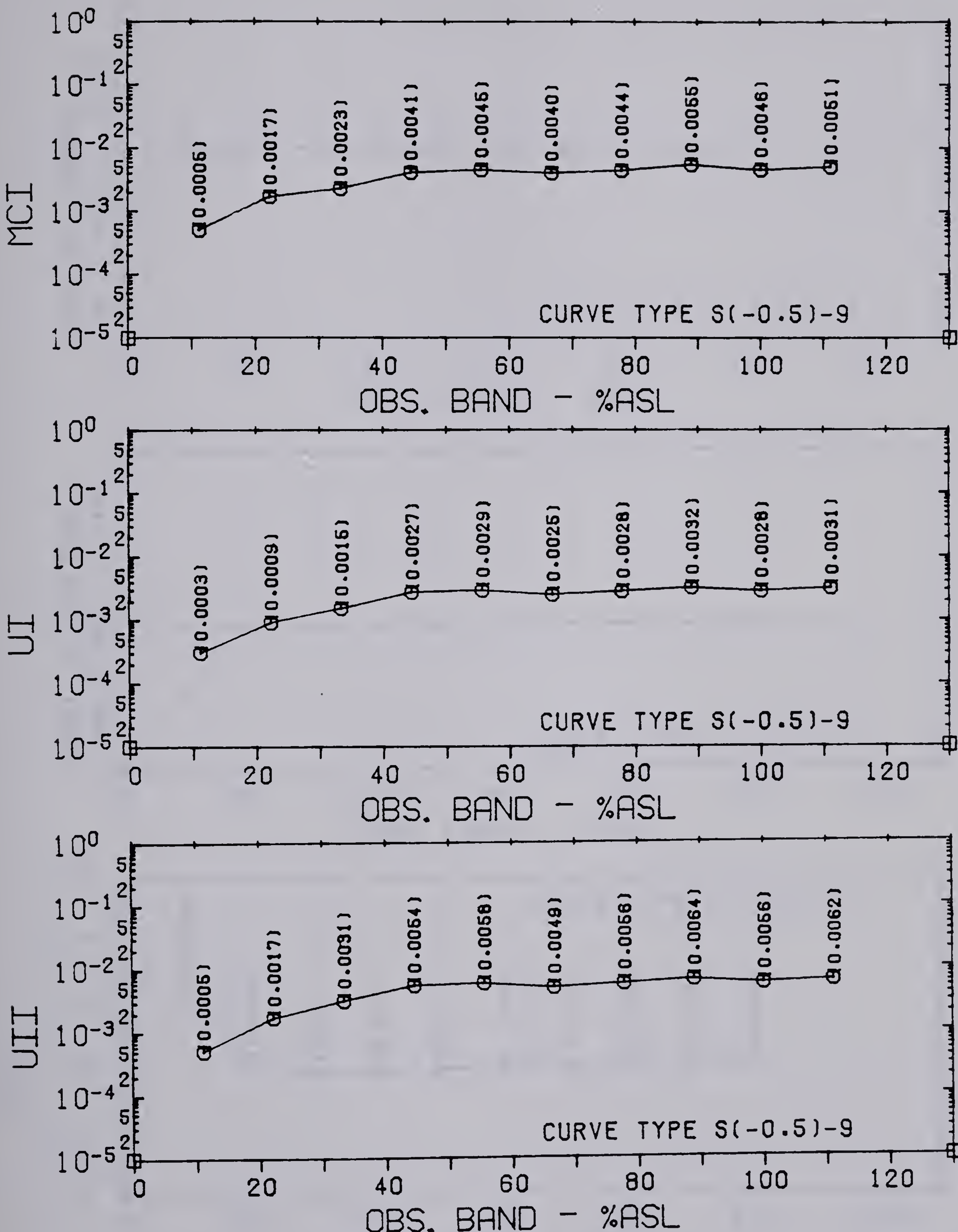


Figure 6.8 Results of the Investigation of the Observation Band Length for a S(-0.5)-9 Curve With an Exponenetial Growth Rate of 1.04



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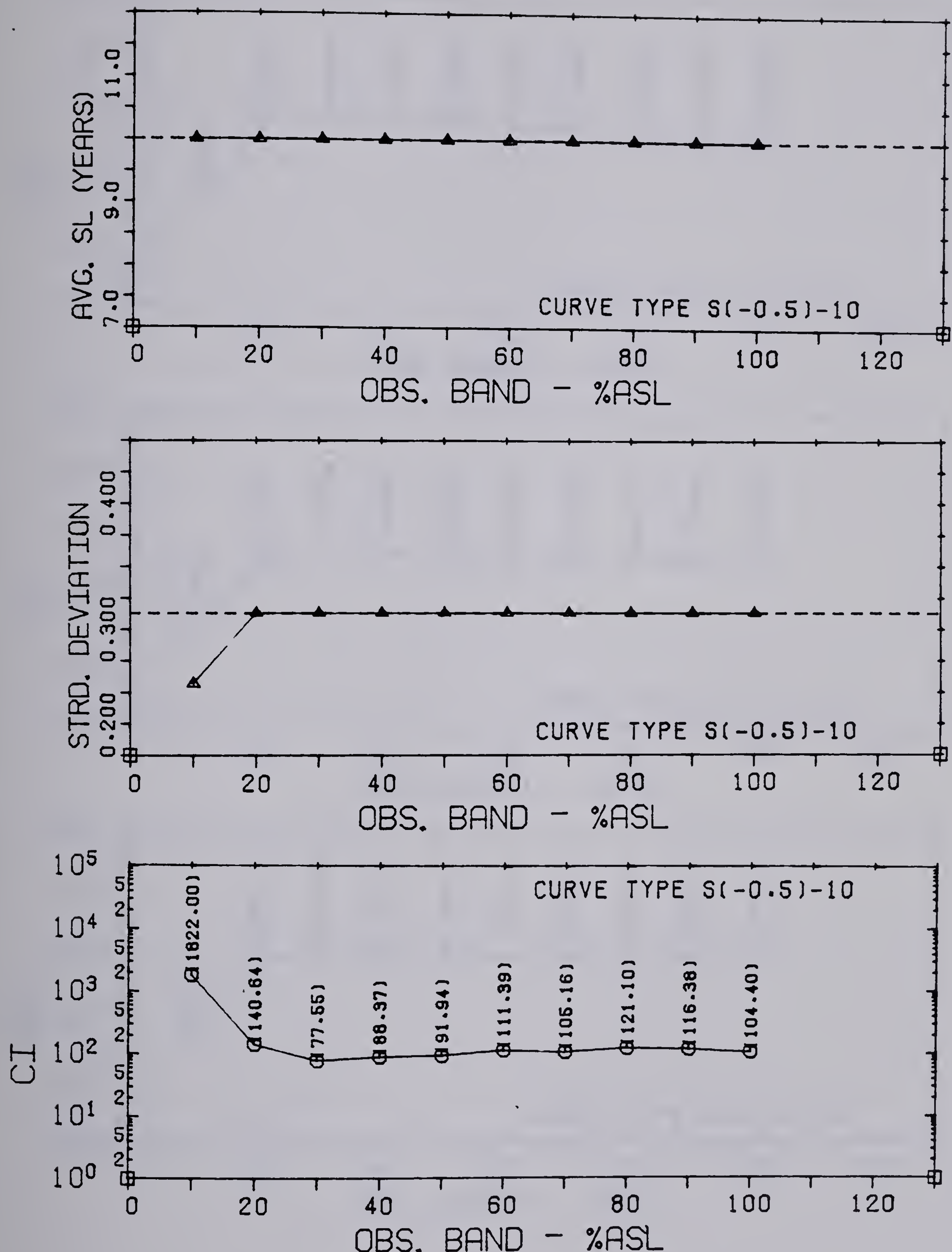


Figure 6.9 Results of the Investigation of the Observation Band Length for a S(-0.5)-10 Curve With a Stationary Plant Balance

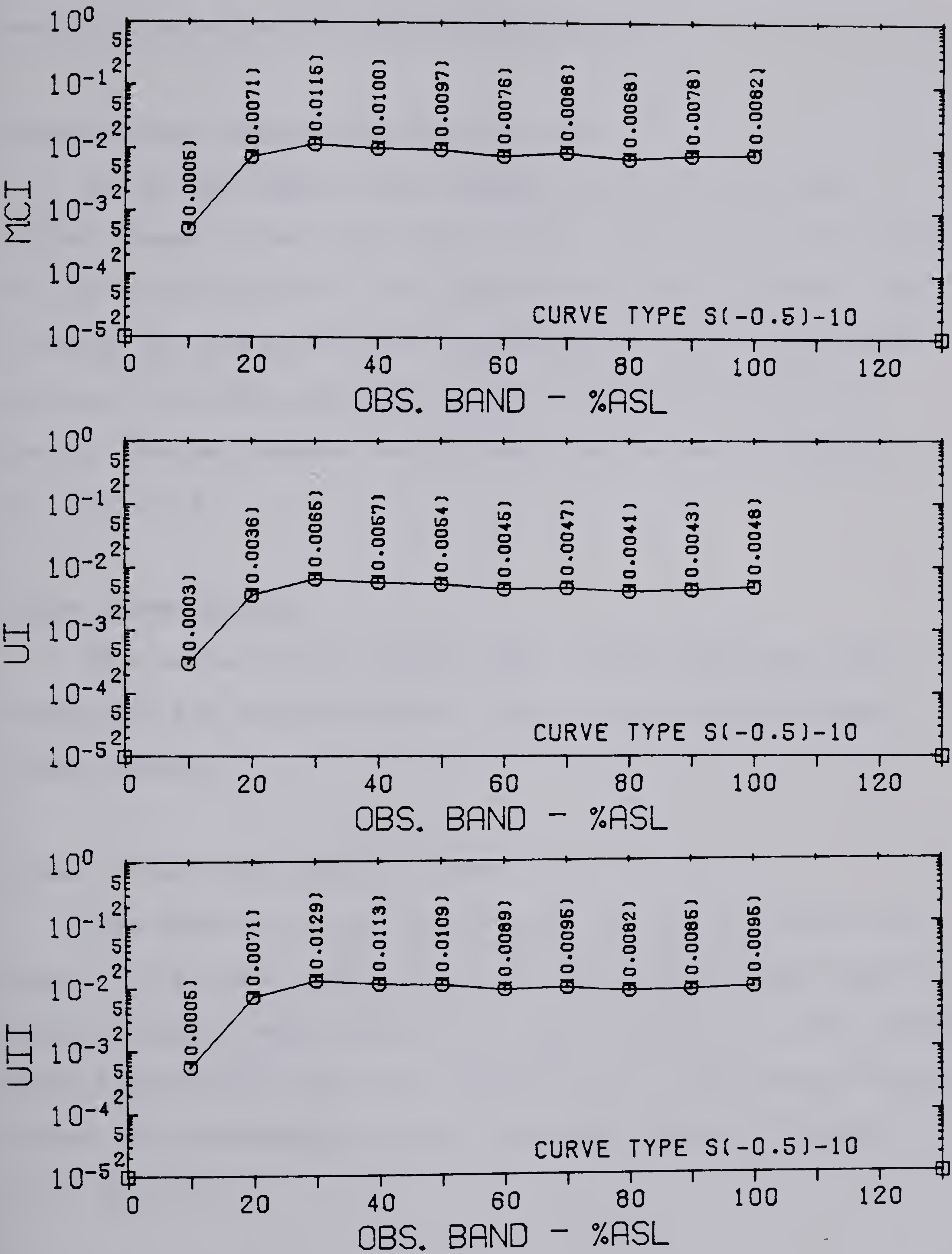


Figure 6.9 Continued from the Previous Page.

length of the Observation Band appears to be about 55% or more of the actual average service life.

Higher Order Symmetrical Modal Curves

As in the case of left modal curves of the higher order, these curves also show a high degree of insensitivity to the quantity of the available actual data. However, the indices for the stationary accounts show very unfavorable values indicating that this kind of account is hard to match. The performance curves have been shown in Figures 6.10 to 6.12.

Right Modal Curves

The tests for the right modal curves also have been conducted for both the lower order curves and the higher order curves.

Lower Order Right Modal Curves

The behavior of the lower order curves is similar to that of the other modal types. The selected average service life has again been found to be less sensitive to the actual data availability than the sensitivity of the selected type curve. The performance curves have been shown in Figures 6.13 to 6.15.

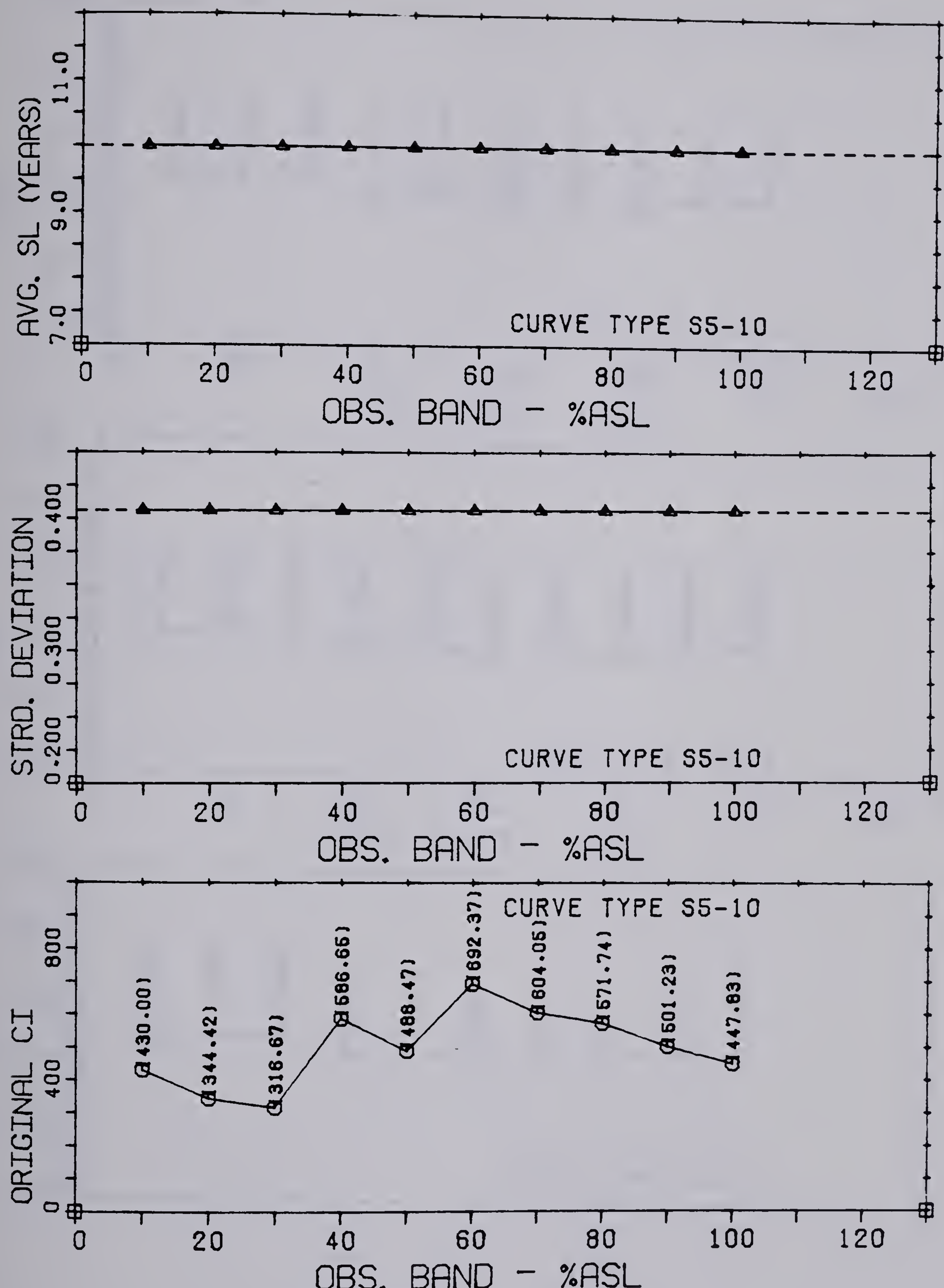
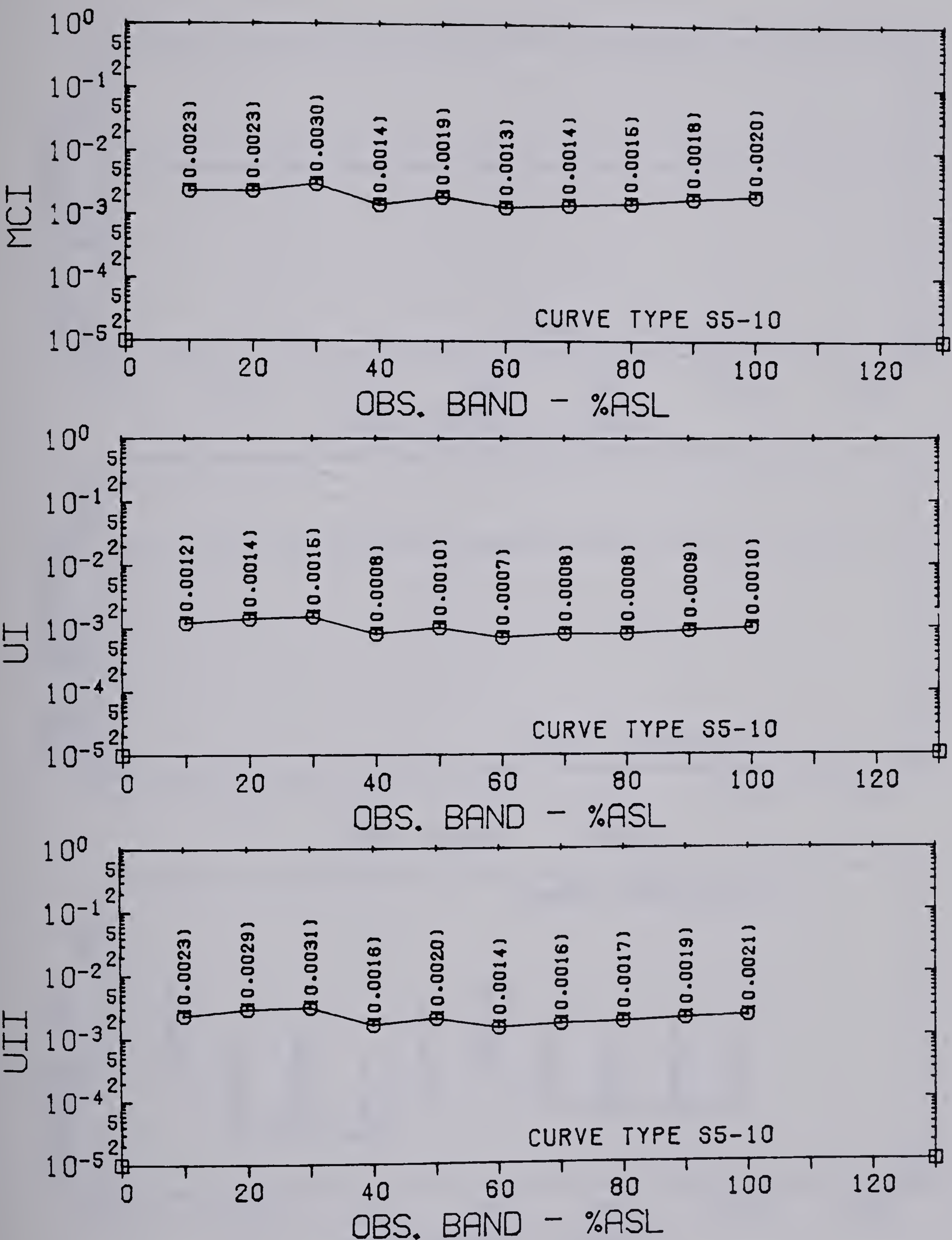


Figure 6.10 Results of the Investigation of the Observation Band Length for a S5-10 Curve With a Linear Growth Rate of 3000 Units/Yr.



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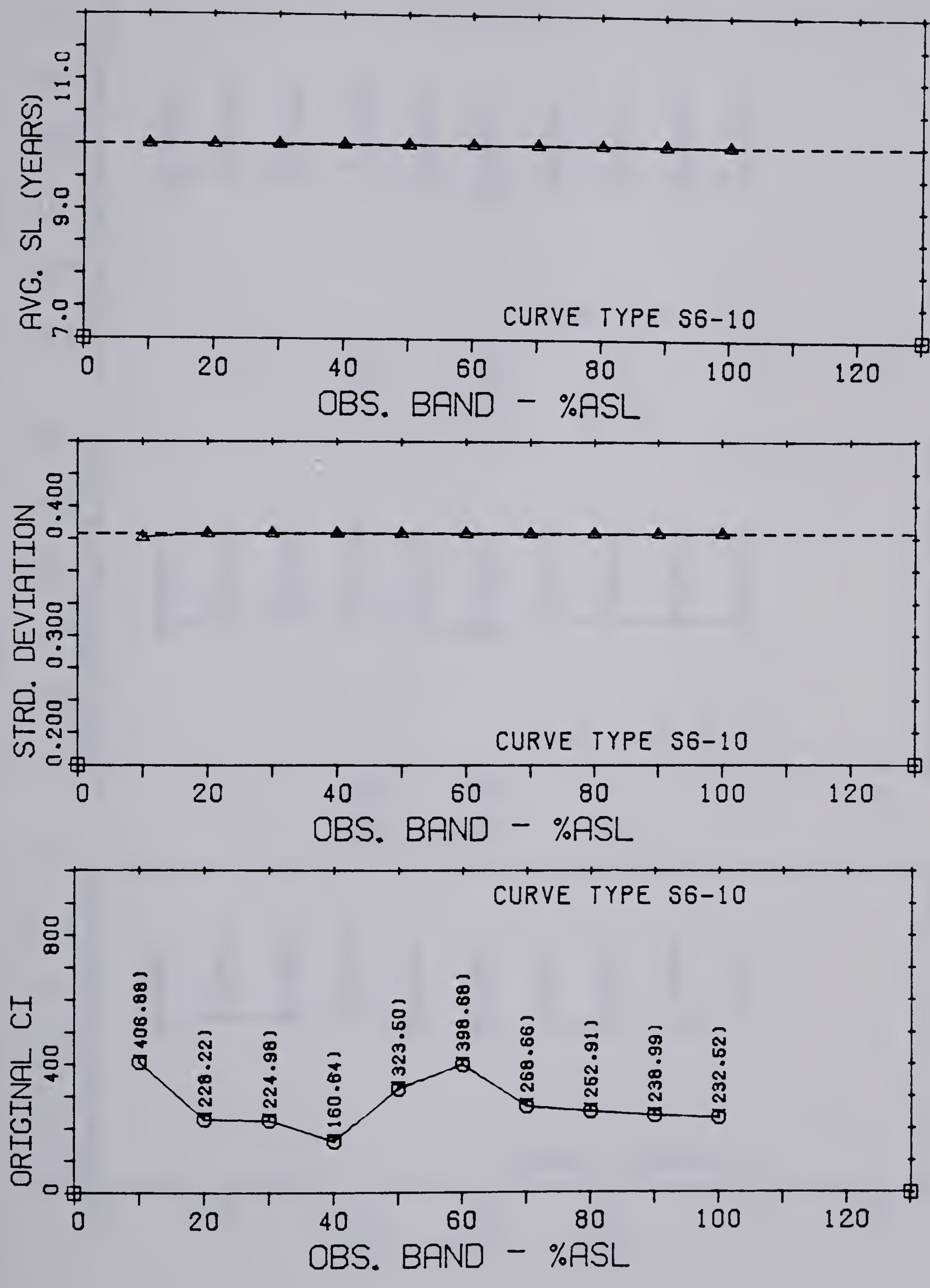
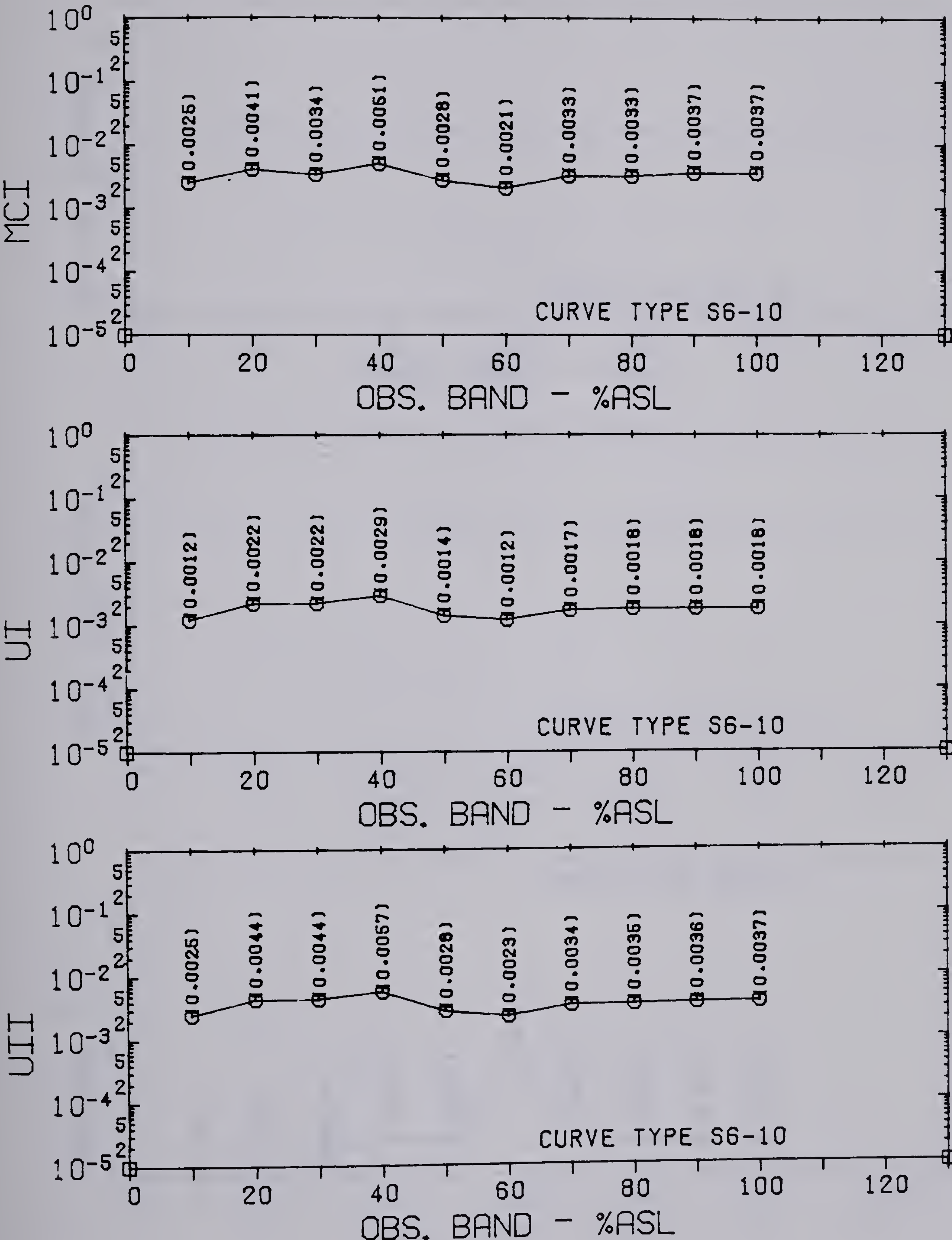


Figure 6.11 Results of the Investigation of the Observation Band Length for a S6-10 Curve With an Exponential Growth Rate of 1.03



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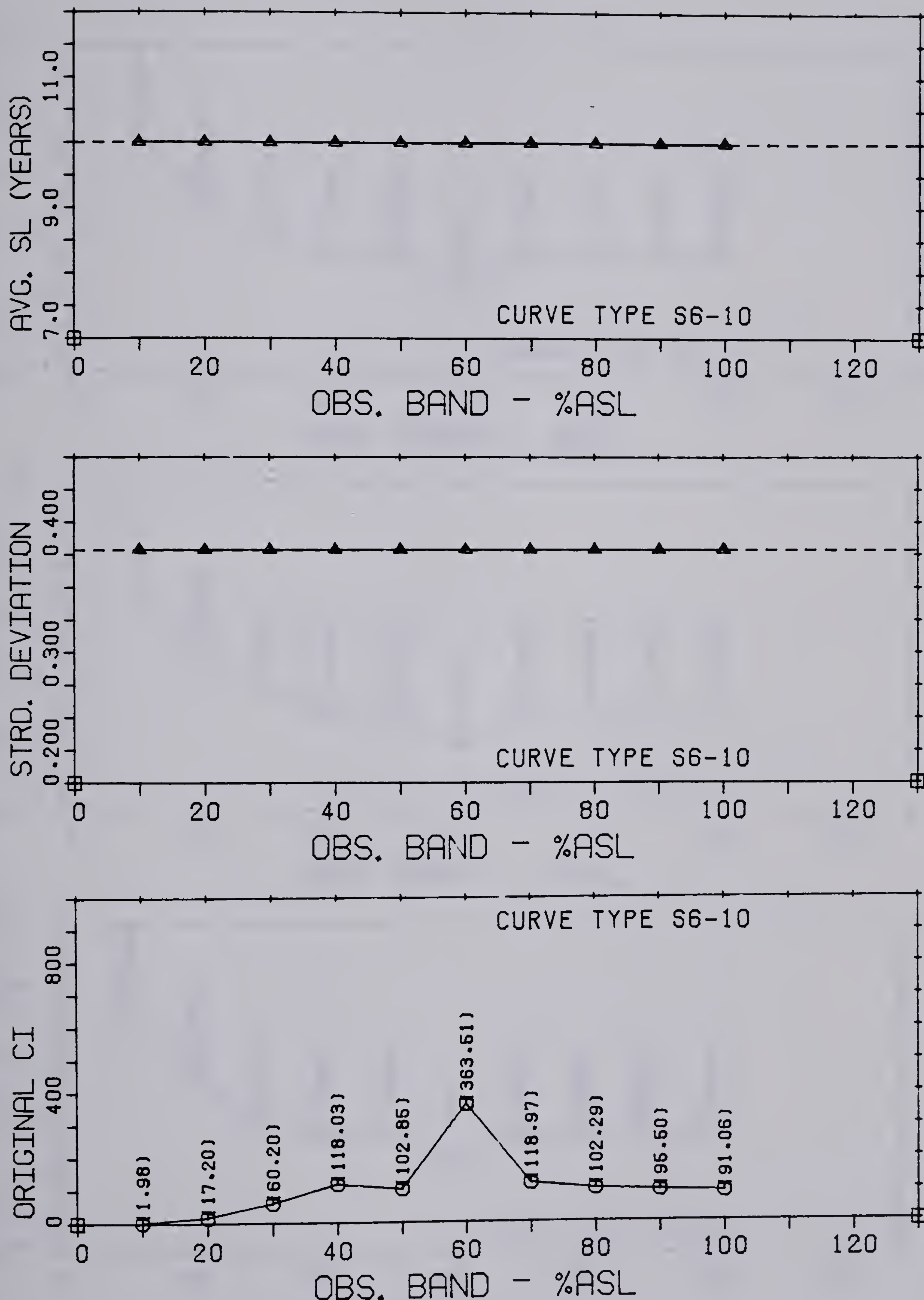
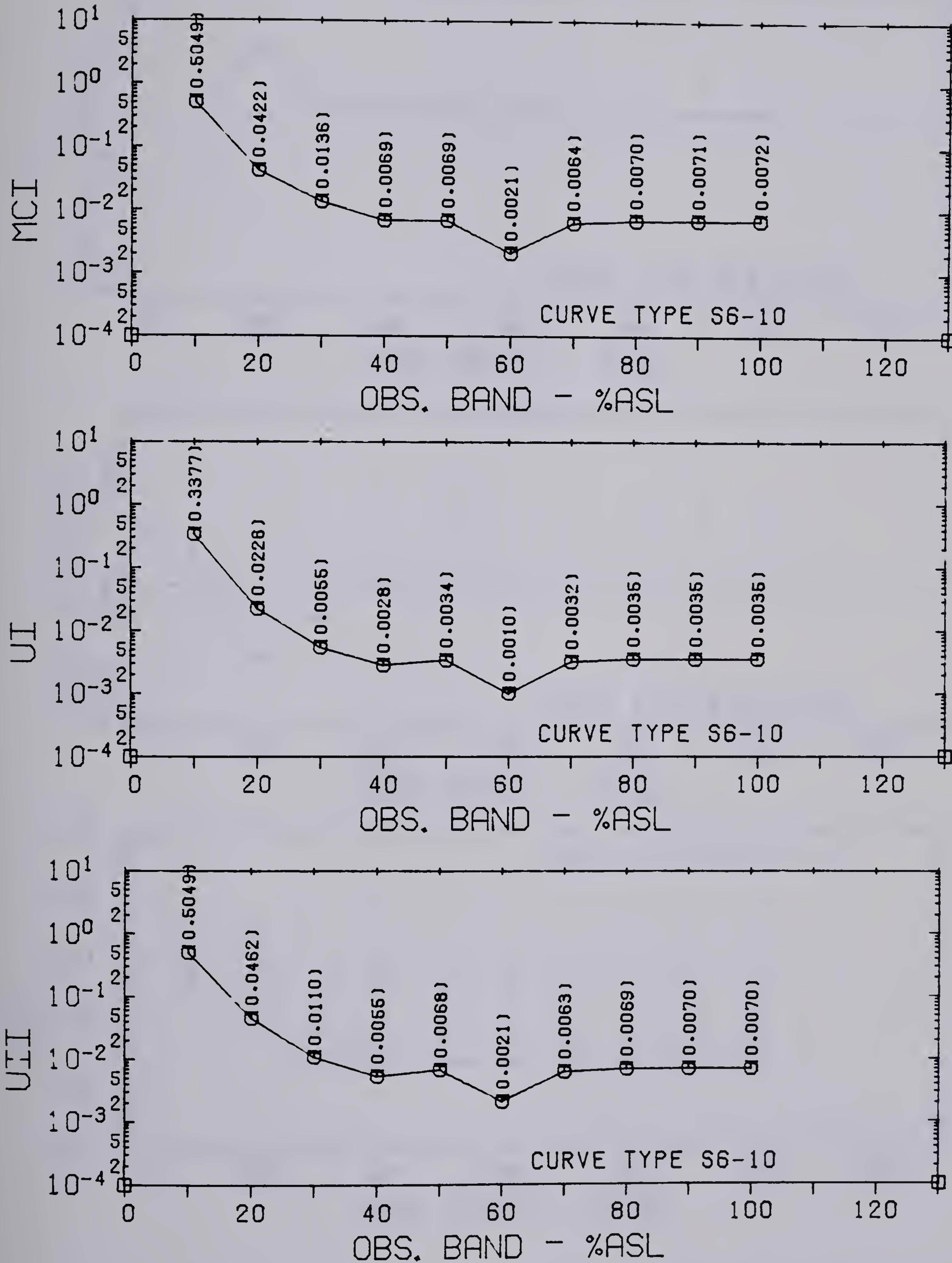


Figure 6.12 Results of the Investigation of the Observation Band Length for a S6-10 Curve With a Stationary Plant Balance



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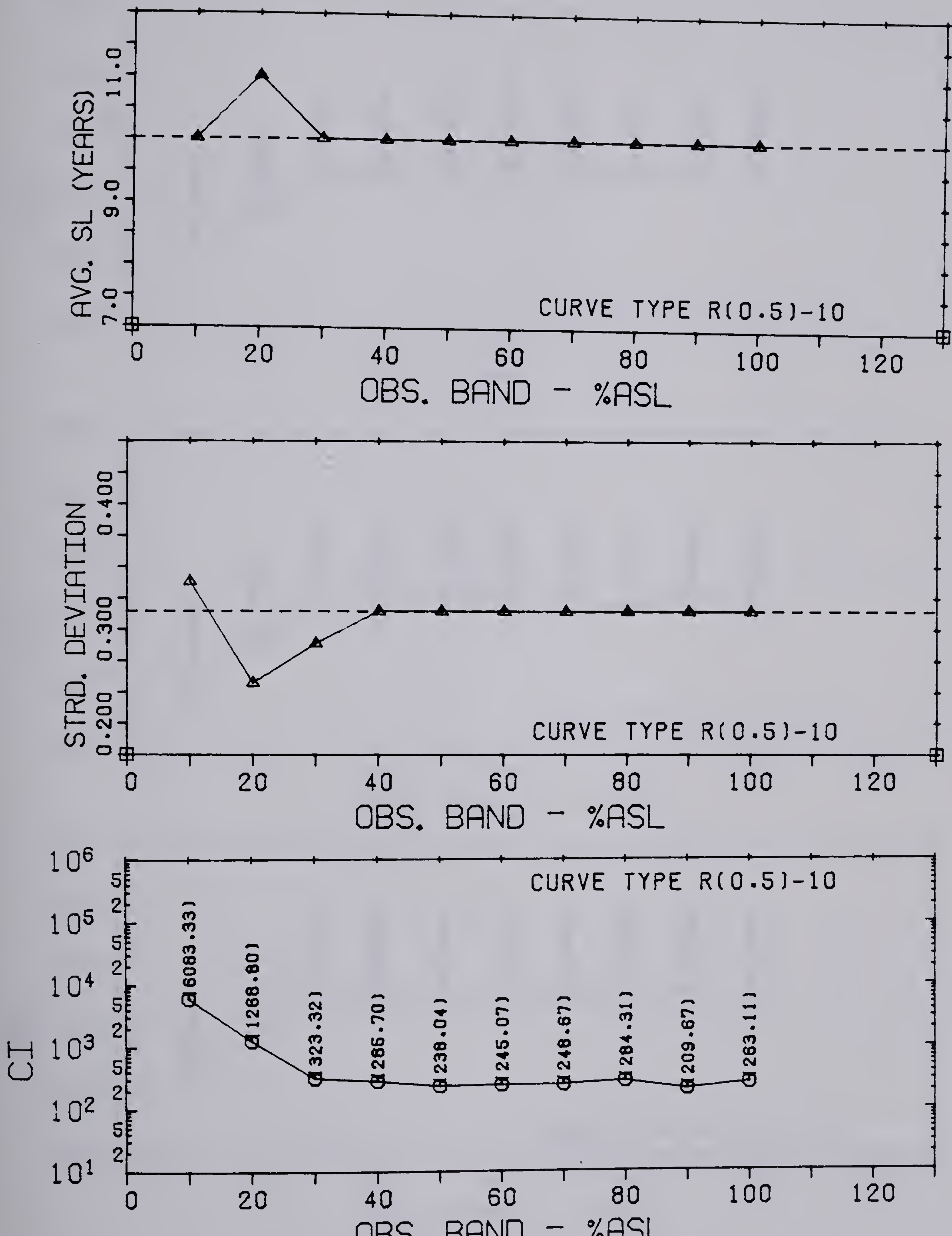
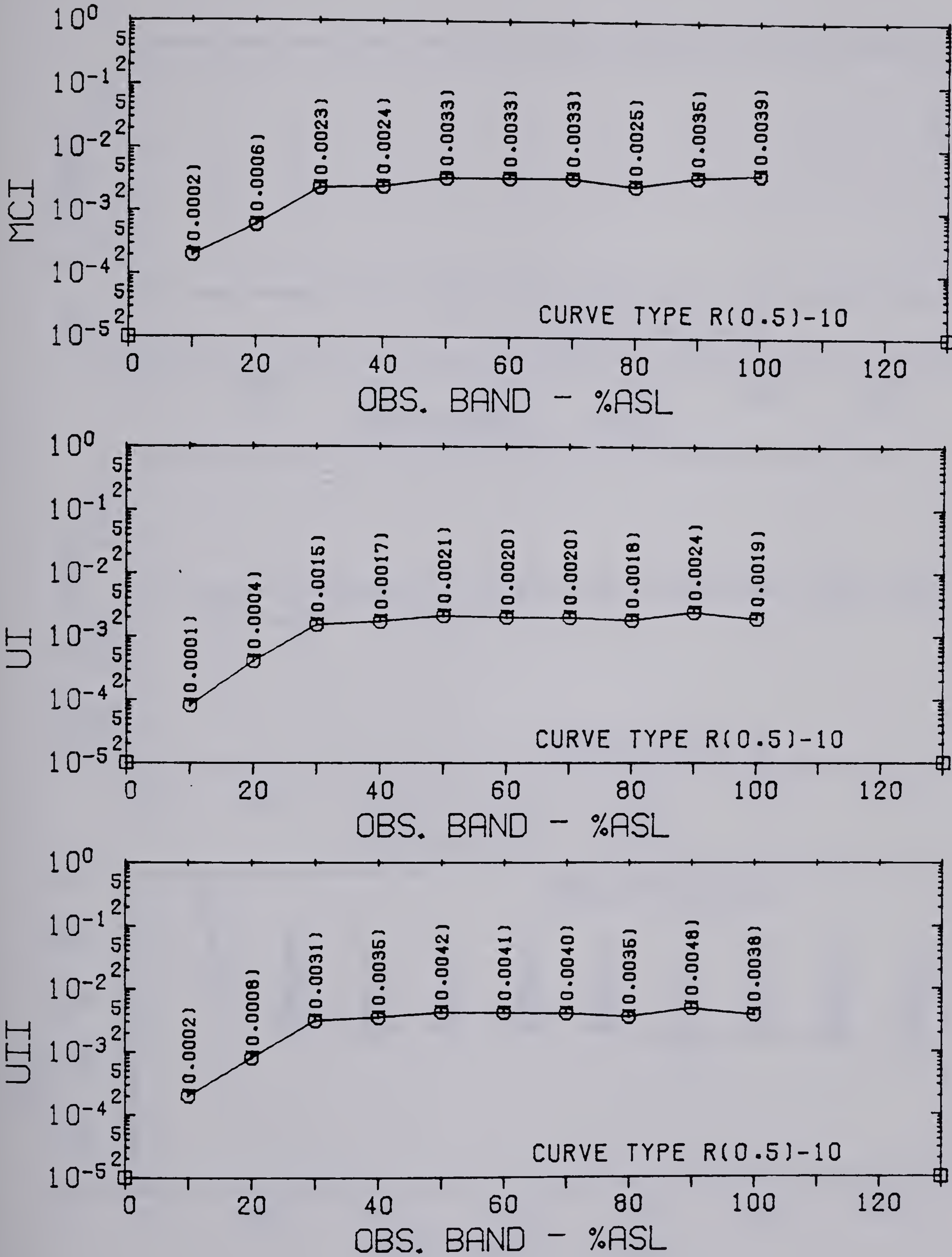


Figure 6.13 Results of the Investigation of the Observation Band Length for a R(0.5)-10 Curve With a Linear Growth Rate of 3700 Units/Yr.



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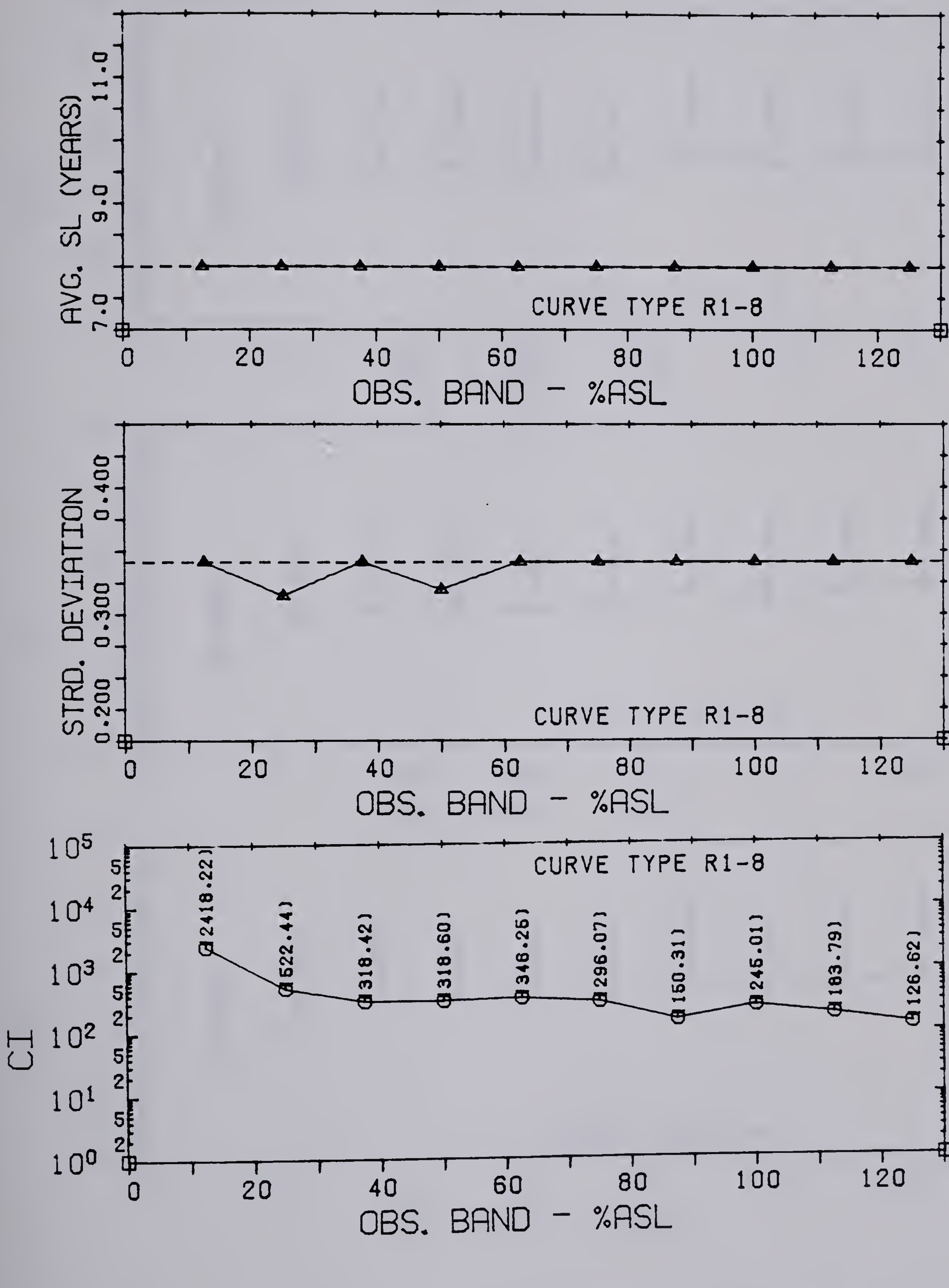
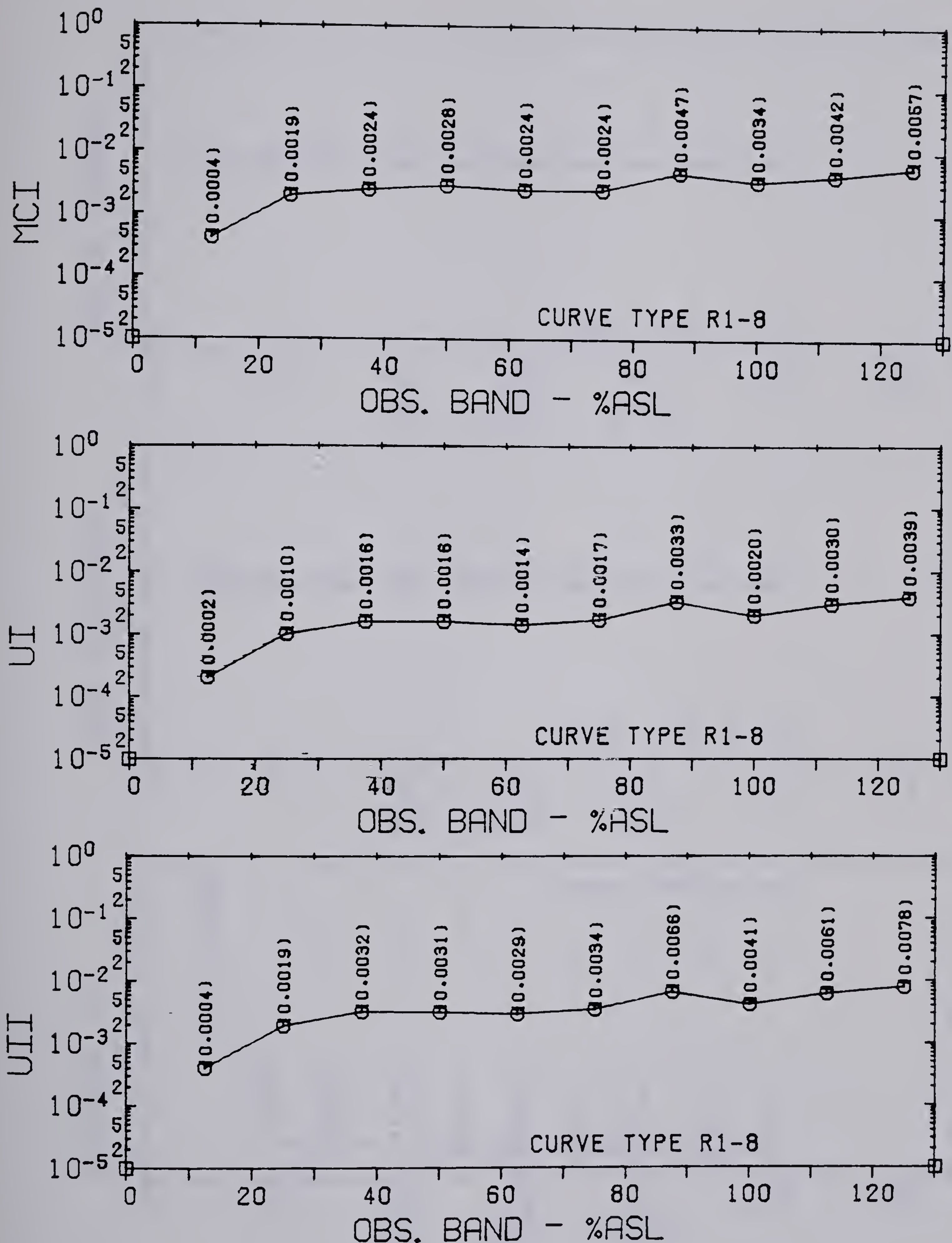


Figure 6.14 Results of the Investigation of the Observation Band Length for a R1-8 Curve With an Exponential Growth Rate of 1.03



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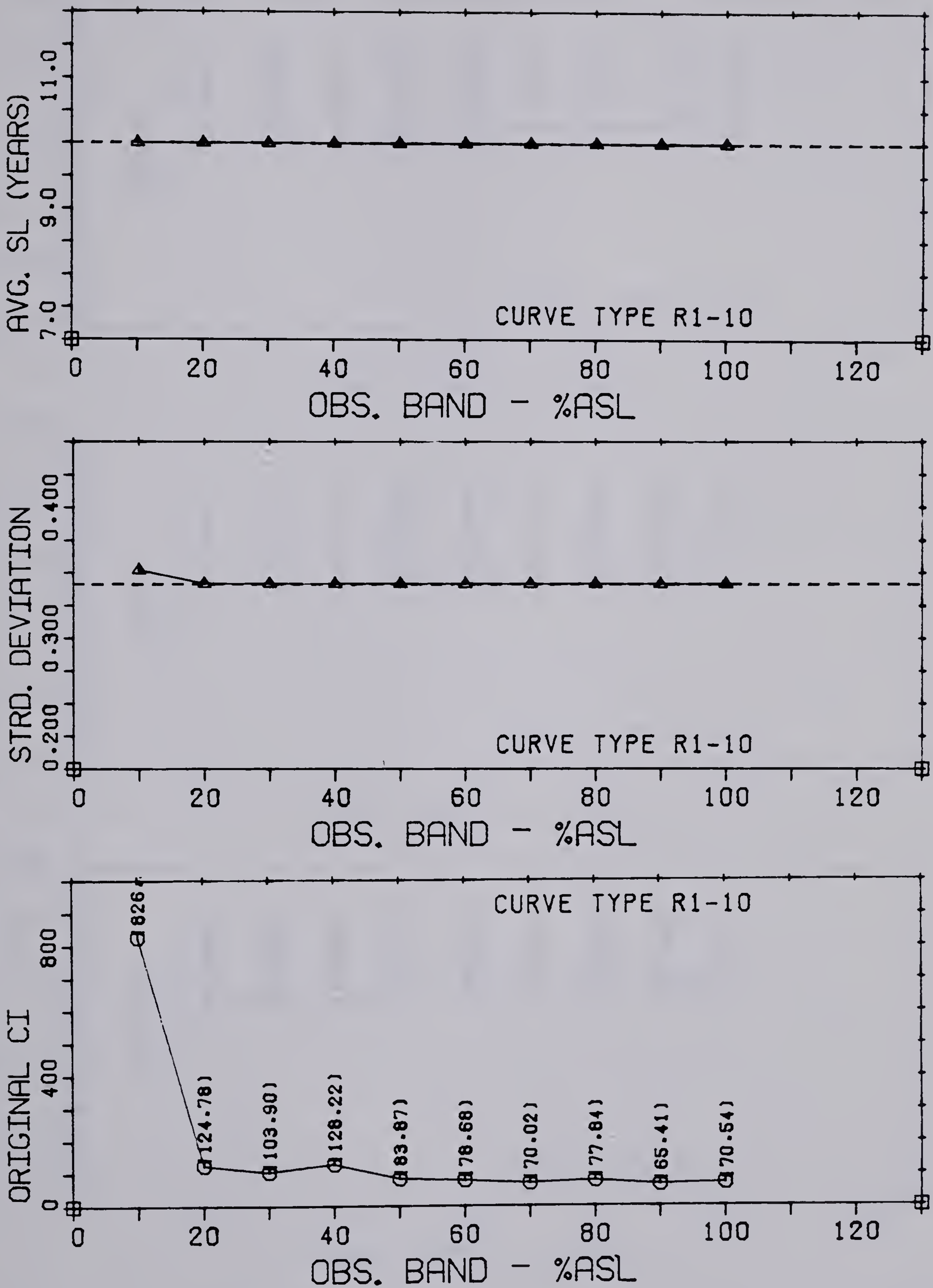
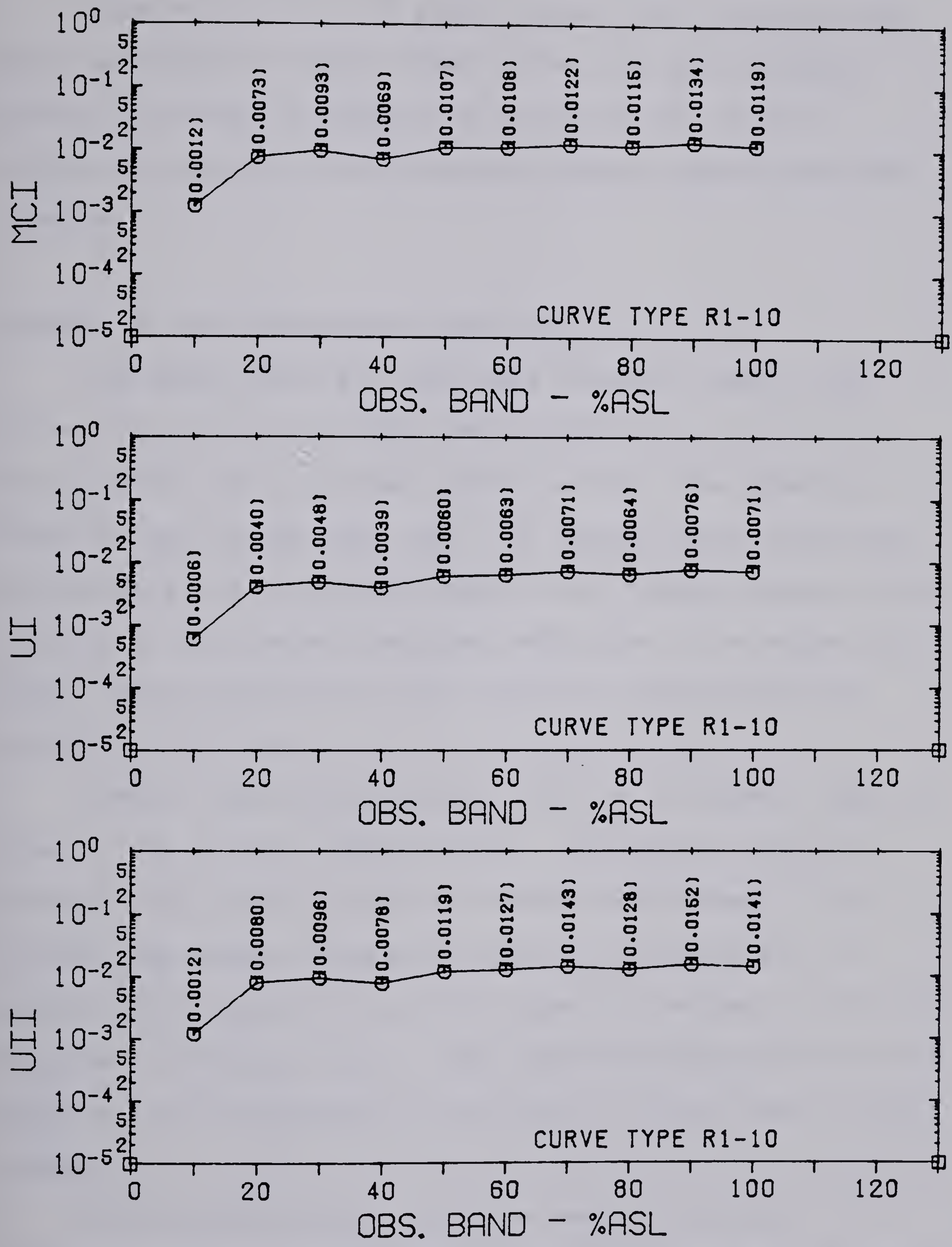


Figure 6.15 Results of the Investigation of the Observation Band Length for a R1-10 Curve With a Stationary Plant Balance



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Higher Order Right Modal Type

Figures 6.16 to 6.18 again suggest that property data sets generated by higher order curves are quite unique. Hence, the model is capable of selecting the correct mortality characteristics even with short Observation Band lengths.

Summary of the Observation Band Tests

The above tests for the three different modal type curves are indicative that lower order curves need more actual data than the higher order curves. This behavior might be due to the fact that, for higher order curves the retirements are clustered close to the average service life. Therefore, this makes the plant additions to be unique and hence fully reflective of the mortality characteristics generating the data.

However, when the available data is very small (50% or less of the average service life), the indices should be treated with caution. This is because the values of the indices may appear favorable inspite of a wrong set of specified characteristics. This aspect is evident in all the Figures 6.1 through 6.18. A more detailed discussion on this behavior of the indices will follow at a later time in this chapter.

The results obtained from the above tests are suggestive that the Observation Band length must be at least 70 to 80% of the average service life in order to have an

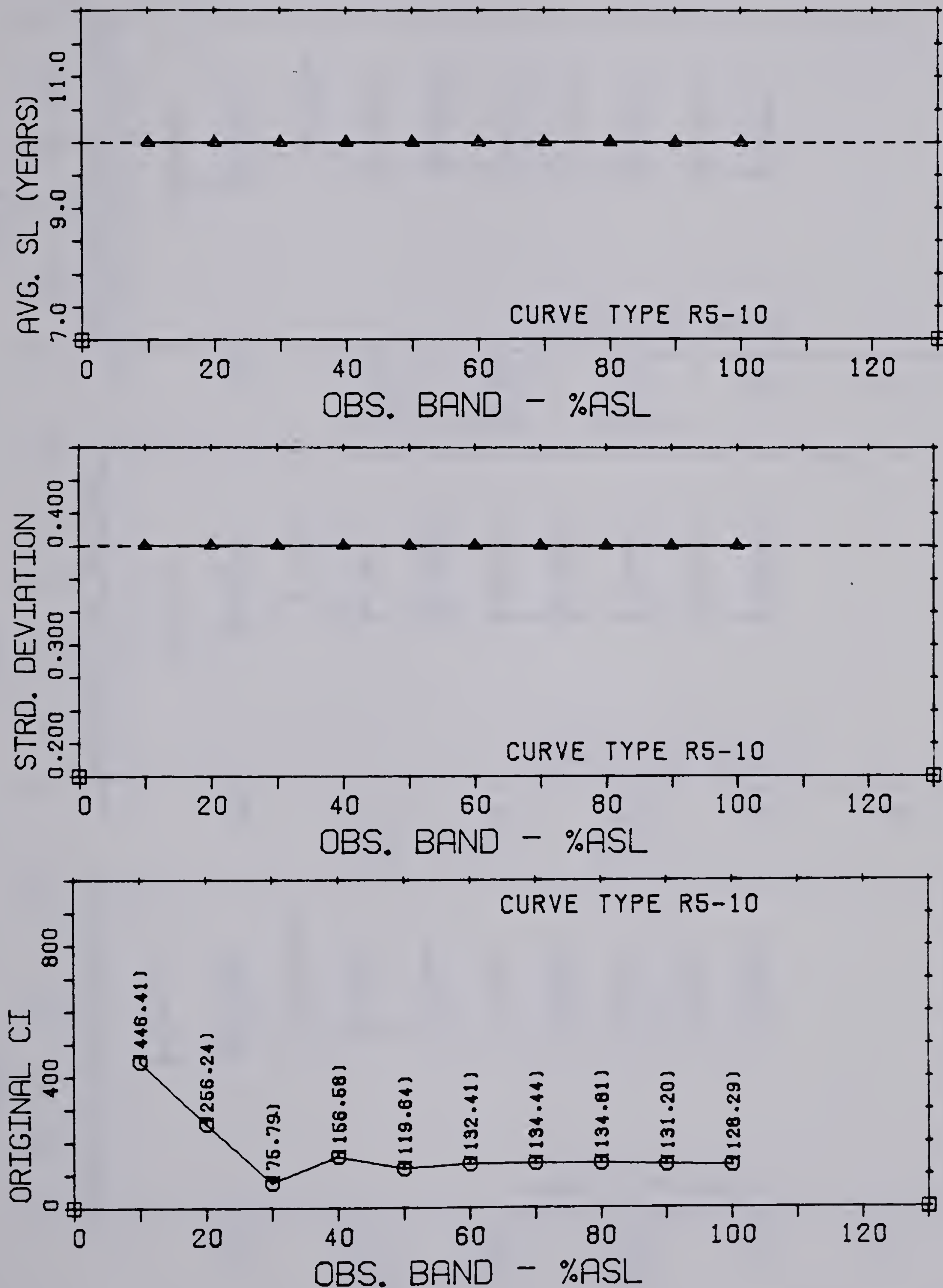
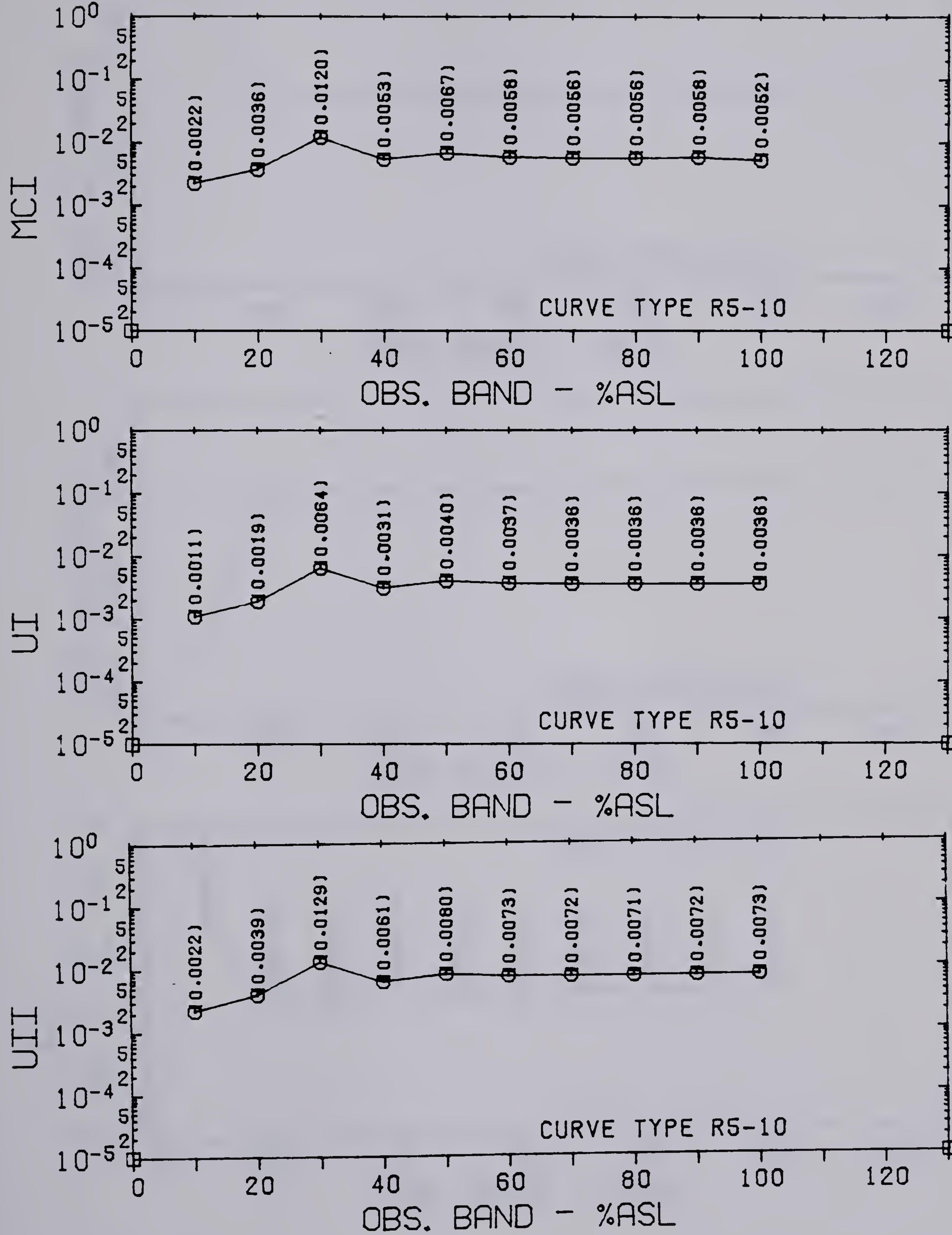


Figure 6.16 Results of the Investigation of the Observation Band Length for a R5-10 Curve With a Linear Growth Rate of 2500 Units/Yr



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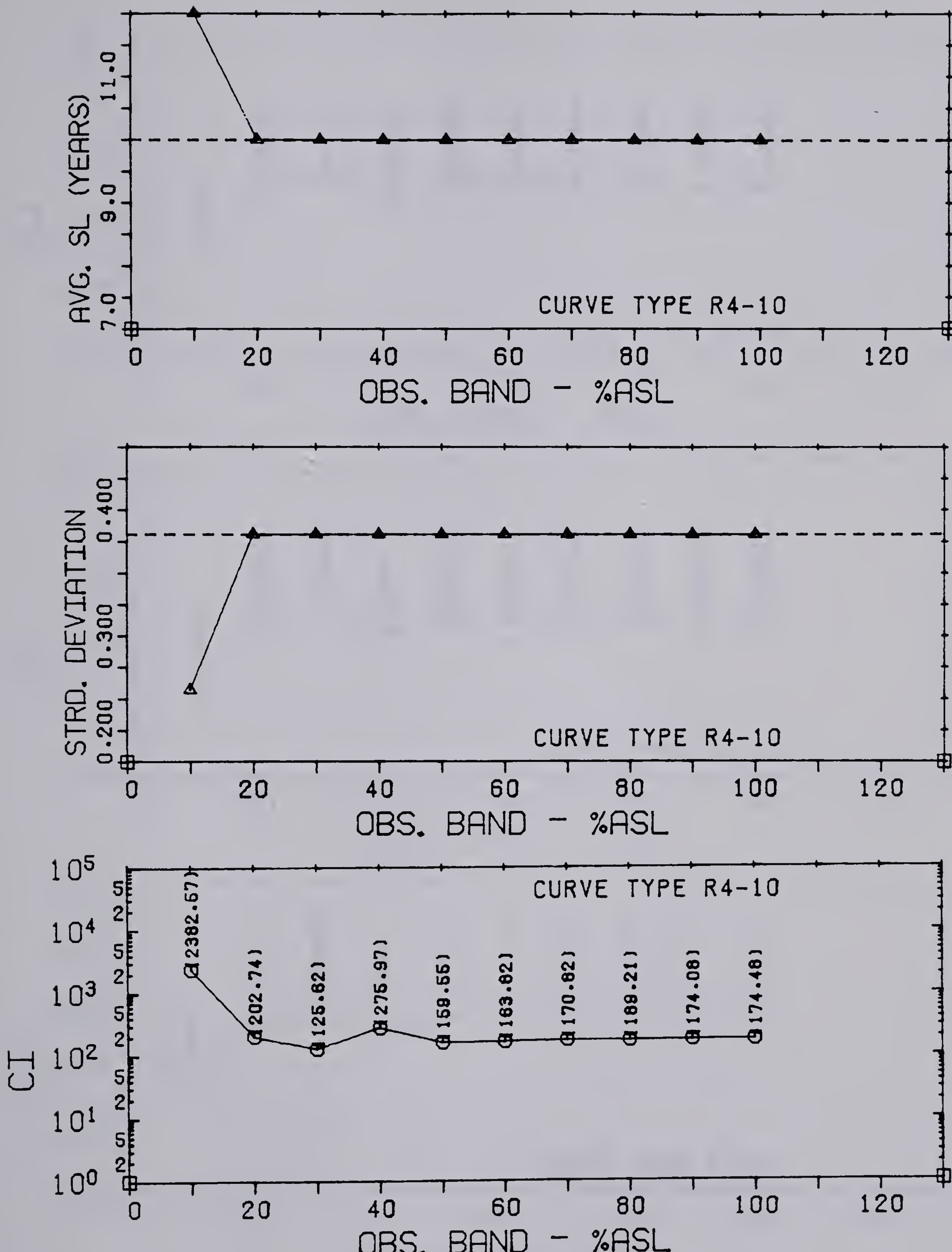
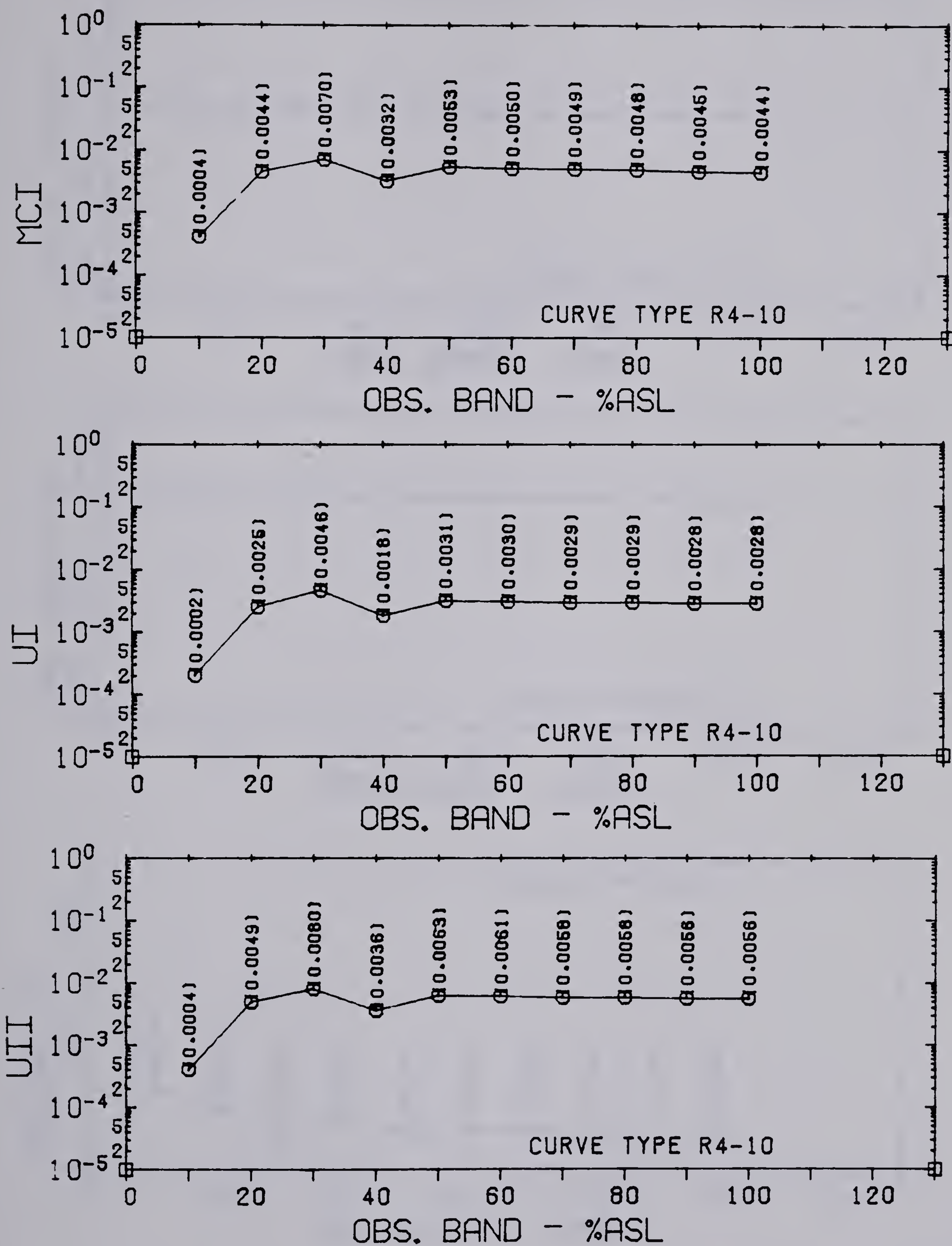


Figure 6.17 Results of the Investigation of the Observation Band Length for a R4-10 Curve With an Exponential Growth Rate of 1.03



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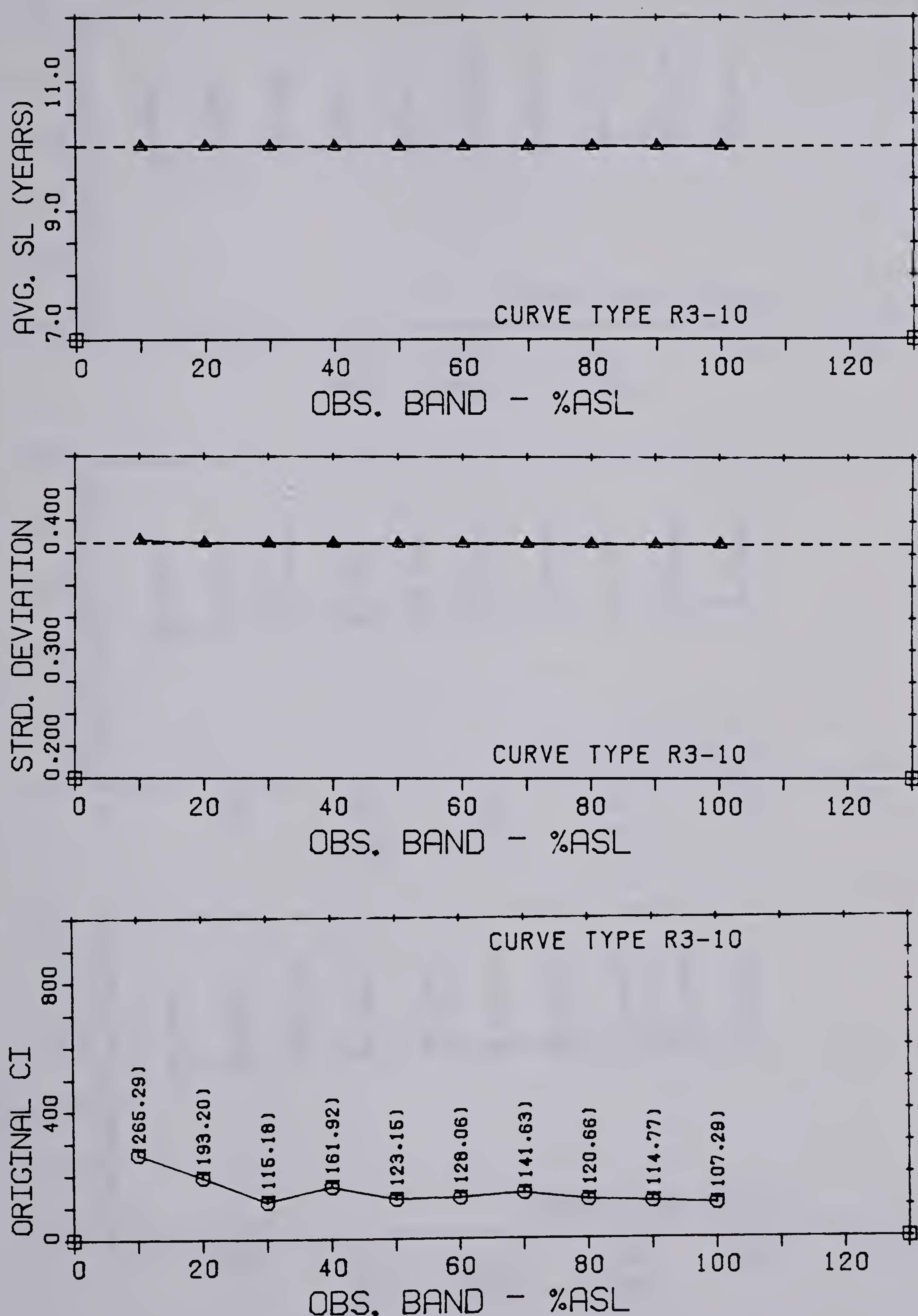


Figure 6.18 Results of the Investigation of the Observation Band Length for a R3-10 Curve With a Stationary Plant Balance

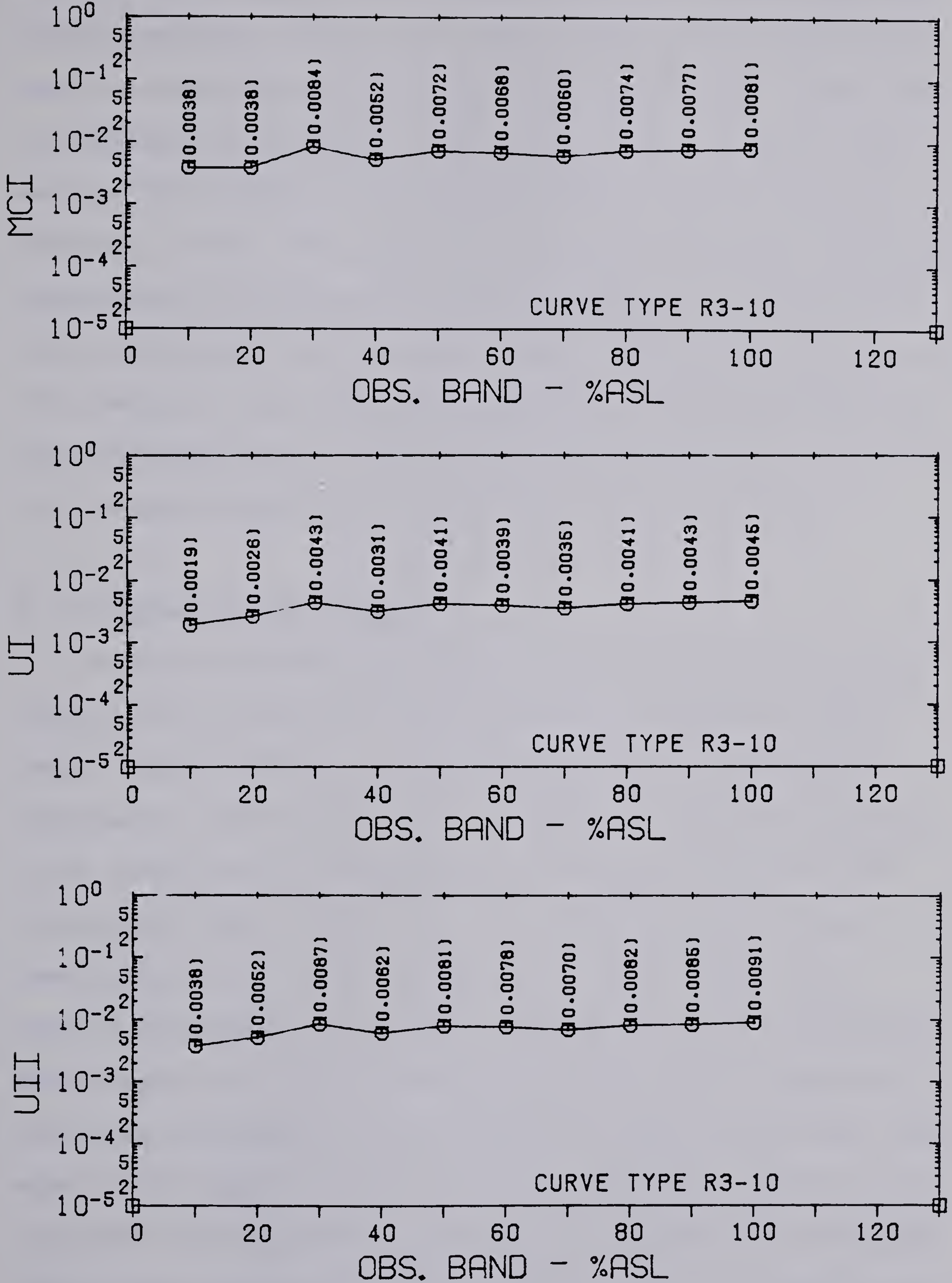


Figure 6.18 Continued from the Previous Page.

effective performance of the model for all curve types and growth profiles. Though the higher order curves apparently require lesser actual data than the lower order curves, it is necessary to provide an Observation Band length of as much as 70% to 80% of the average service life. This is because, in real life applications, the curve type which is generating the retirements will be unknown to the analyst (the curve type will be one of the variables being sought by the analyst). Such being the case, it will be essential to provide enough data (70% to 80% of the average service life) for the most demanding type curves to be satisfied.

6.3 Transparent Band tests

After the Observation Band length, the next parameter required to be tested is the specified Transparent Band length and its effect on the final result. Although the Transparent Band length will be generally a known parameter, it is likely that occasionally an accurate length of the Transparent Band length (ie. how old the account under consideration is) will be unknown. In such cases, the analyst will have to specify an estimate of the Transparent Band length for the test which may be in error compared to the actual Transparent Band length. If this is the case, the model might behave differently than expected. Hence, it is important to study and understand the behavior of the model under such circumstances. With this in view, the model was tested to find its sensitivity for varying Transparent Band

lengths. The performance curves (Figures 6.19 to 6.27) have been plotted against the specified Transparent Band length expressed as a percent of the actual Transparent Band length (rather than expressed as a percent of the average service life). This is because, in such cases, any changes in the behavior of the model will be due to the error introduced by the specified Transparent Band length that differs from the actual Transparent Band length.

The following abbreviations are used in addition to the ones already mentioned:

TR.BD. - Specified Transparent Band Length

ACTTBL - Actual Transparent Band Length

A total of 9 different data sets were tested. These data sets were simulated using middle order L,S and R type curves. For each of these curve types, three data sets were simulated; one each of no-growth, linear and exponential growth types. All the data sets had an average service life of 10 years except for one set which had an average service life of 9 years (S1.5-9 linear growth). An Observation Band length of 8 years was provided (as determined in the previous phase of the test - 80% of the ASL of 10 years). The actual Transparent Band length for all the data sets was 17 years (ie. data sets were simulated for 25 years and the data for the last 8 years was treated as the actual data input for the model). This Observation Band length was provided to minimize any distortions that might be otherwise imparted by an unfavorable Observation Band length.

Left Modal Curves

Figures 6.19 to 6.21 show the behavior of a left modal curve of the respective growth profiles and growth rates mentioned on the figures. All three growth profiles were tested with a L1.5-10 curve. It was found that for both linear and exponential profiles (Figures 6.19 and 6.20), the model provided satisfactory results when the specified Transparent Band length was about 55% or higher of the actual Transparent Band length. The indices were highly suggestive of the presence of a wrong parameter when the specified Transparent Band length differed substantially from the actual Transparent Band length. As the specified Transparent Band length was brought closer and closer to the actual Transparent Band length, the indices improved gradually hinting at a more and more favorable combination of the input parameters. However, the performance of the model for a stationary data set (Figure 6.21) was unsatisfactory. Although the selected average service life was correct even when the specified Transparent Band length was about 10% of the actual Transparent Band length, the selected curve did not match the actual type curve even at 100% of the actual Transparent Band length. A L2-10 curve was selected (at about 83% of the actual Transparent Band length) which is a similar and a very close curve to a L1.5-10 type curve. Hence, this error in selection might be just due to the stochastic nature of the data set (ie. because of the stochastic scatter, the specific data set

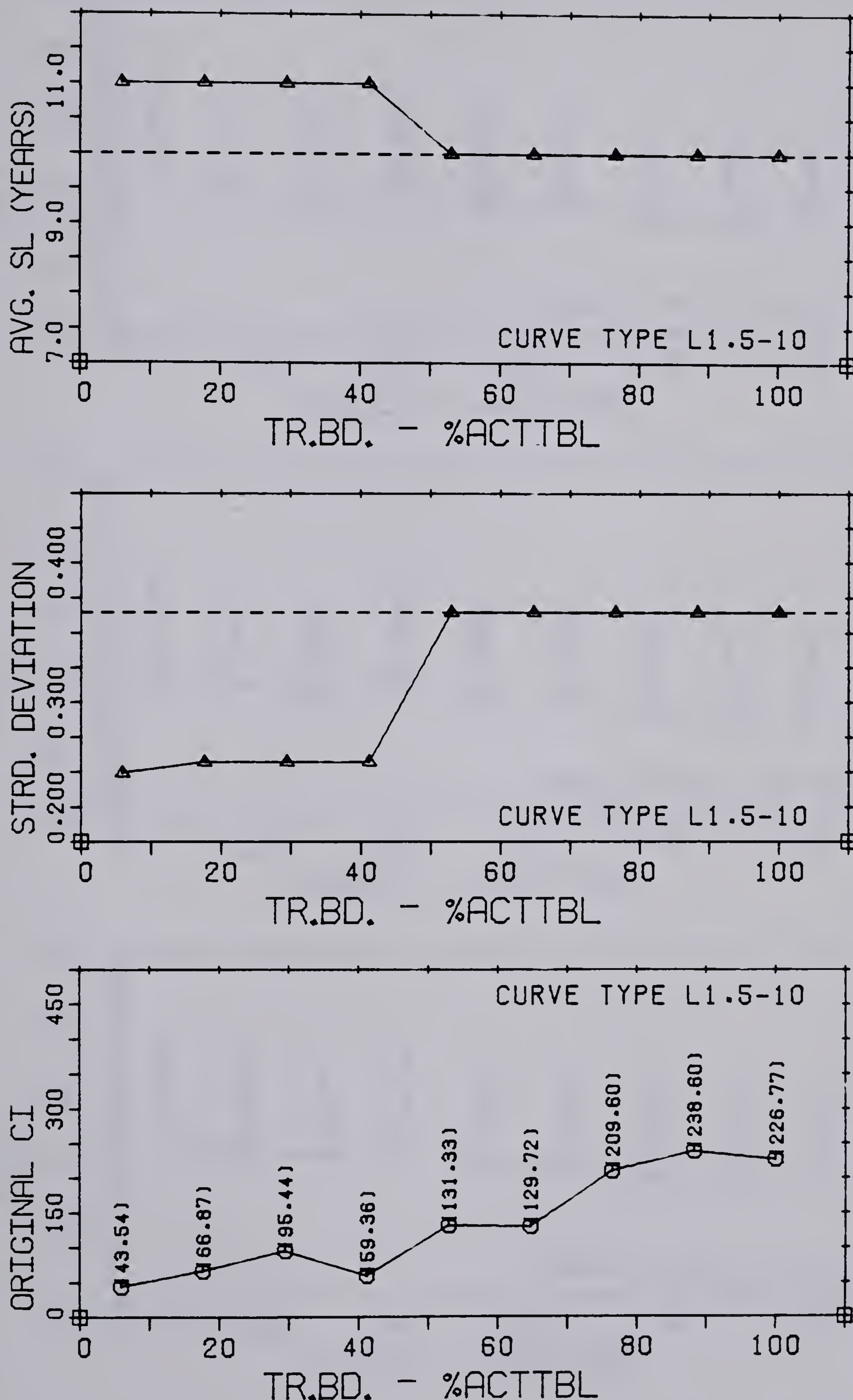
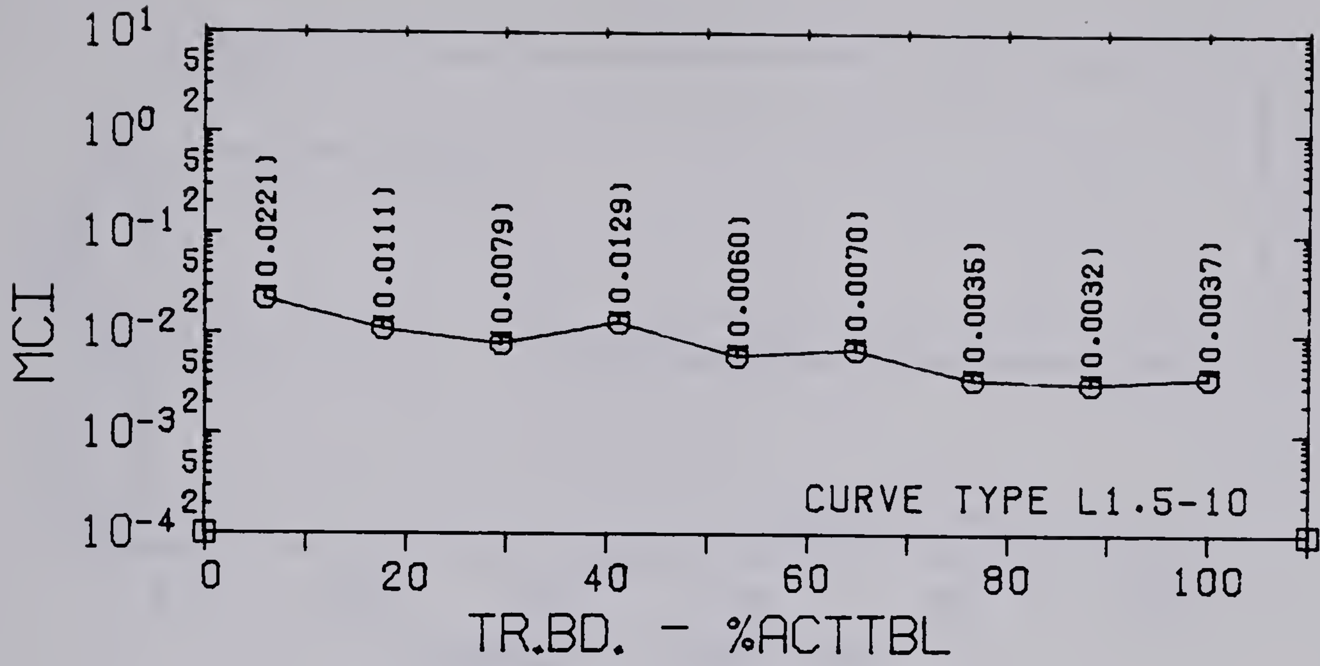
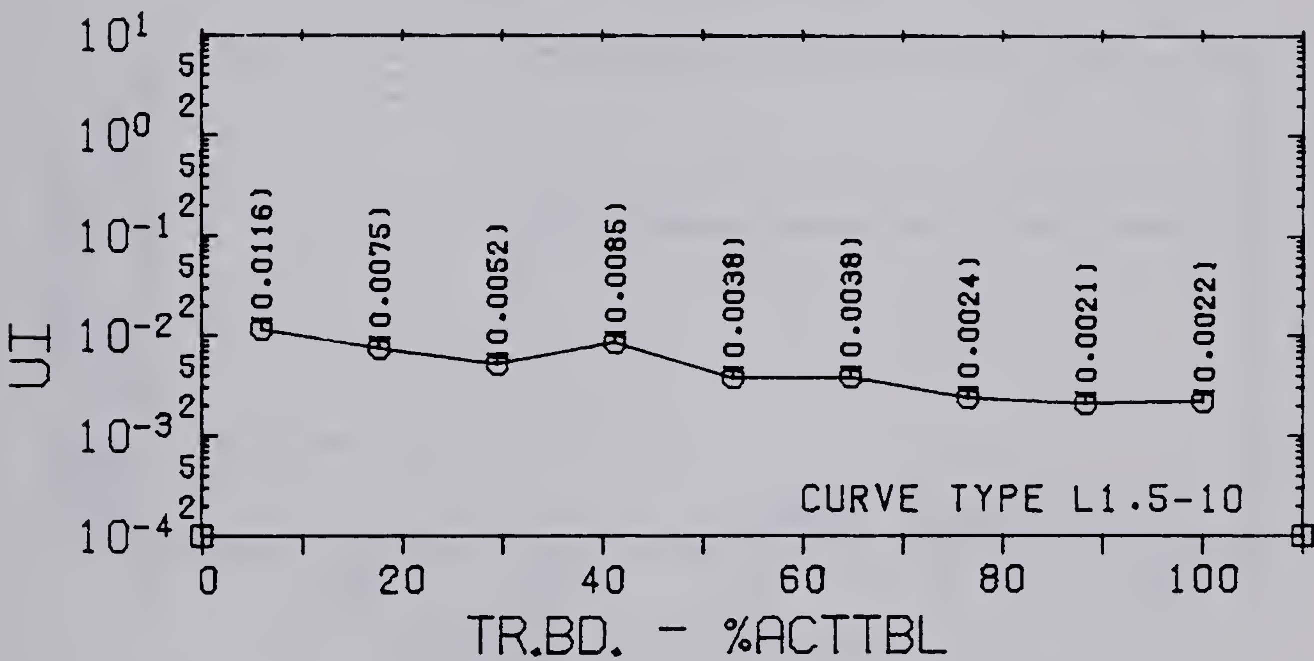
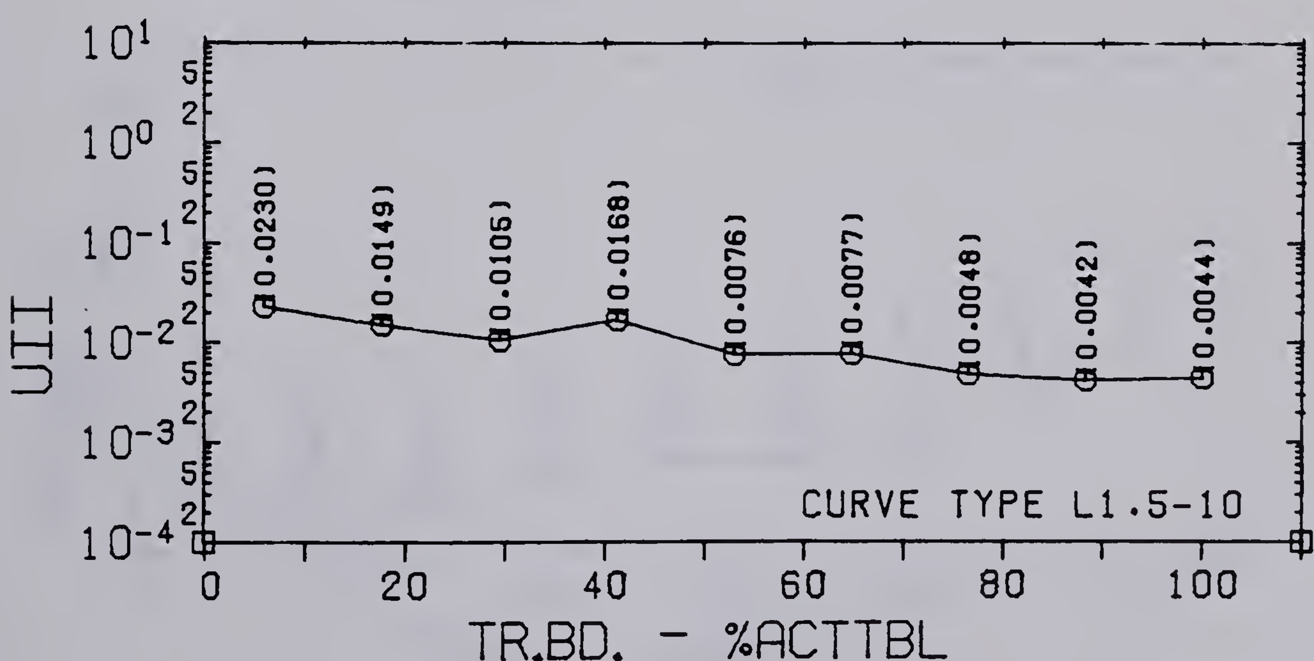


Figure 6.19 Results of the Investigation of the Transparent Band Length for a L(1.5)-10 Curve With a Linear Growth Rate of 2200 Units/Yr.







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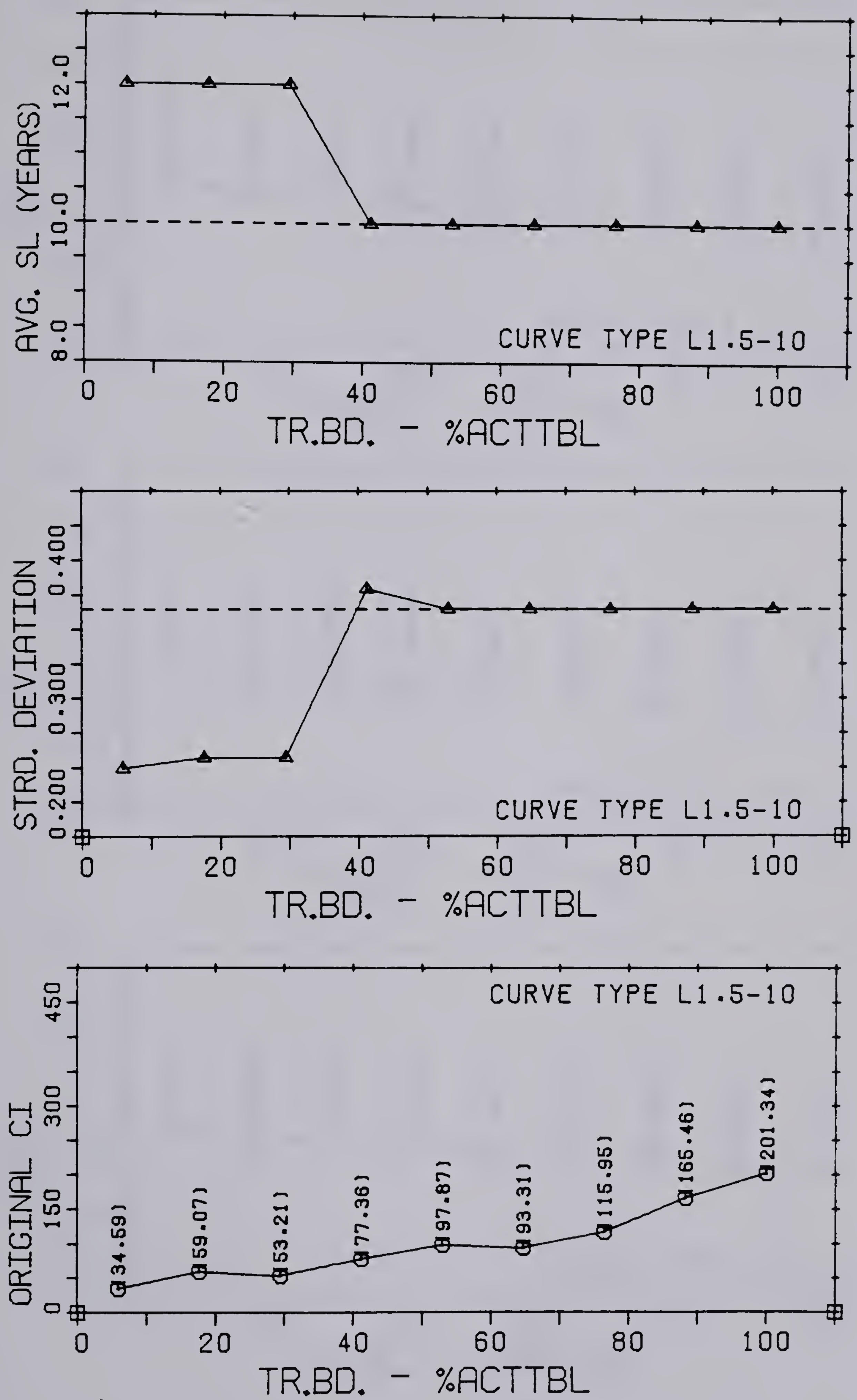
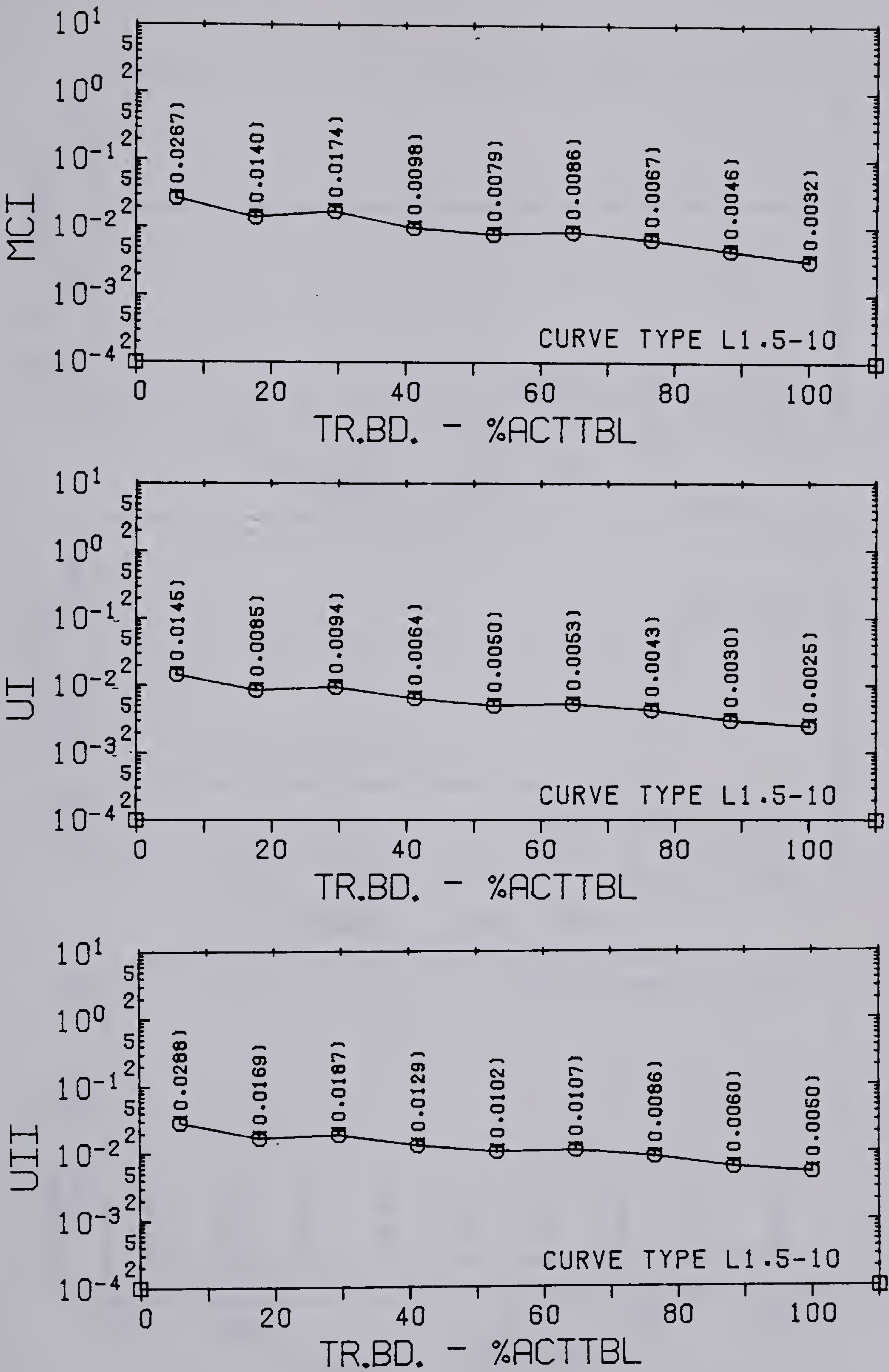


Figure 6.20 Results of the Investigation of the Transparent Band Length for a L(1.5)-10 Curve With an Exponential Growth Raye of 1.04



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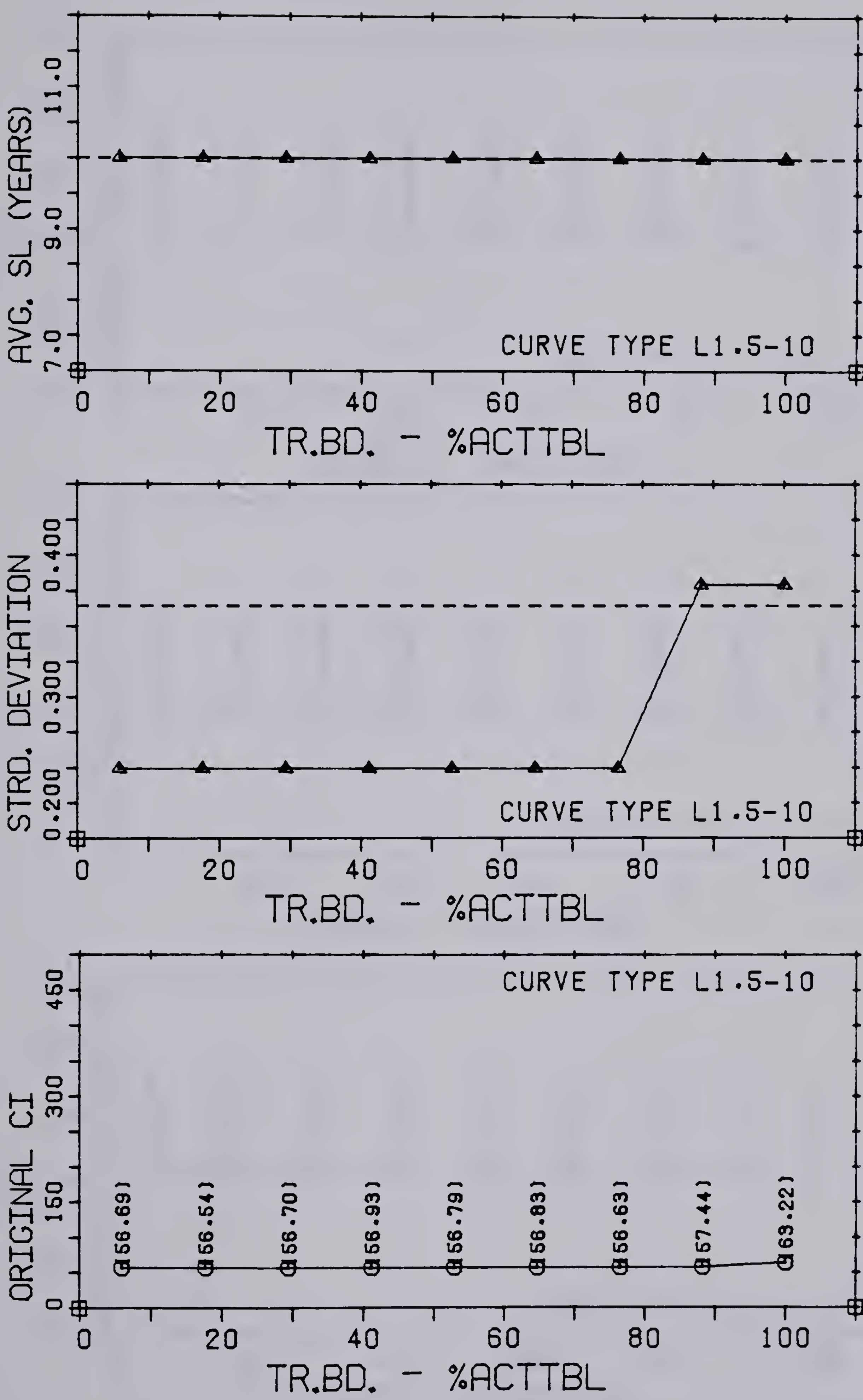
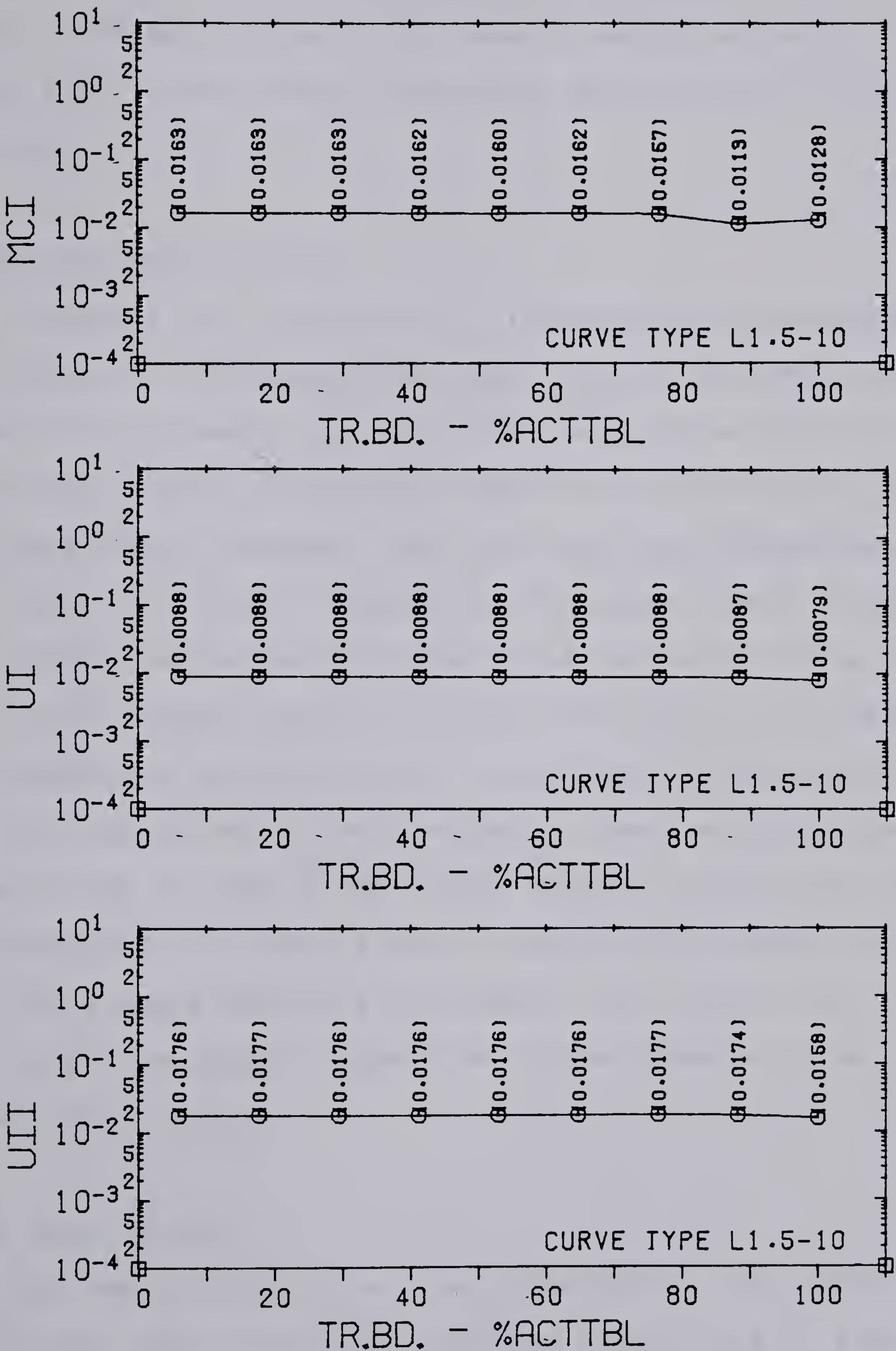


Figure 6.21 Results of the Investigation of the Transparent Band Length for a L(1.5)-10 Curve With a Stationary Plant Balance



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under consideration might be closer to a L2-10 curve than to a L1.5-10 curve). Again, the specified Transparent Band length necessary to yield good results was quite close (about 85%) to the actual Transparent Band length of the data set.

Symmetrical Modal Curves

Figures 6.22 to 6.24 are the results of the Transparent Band tests for the symmetrical modal curves. The performance curves for this modal type are highly suggestive that for a smaller than actual Transparent Band length, the curve type is indeterminate. However, when the specified Transparent Band length is close to the actual Transparent Band length, the standard deviation of the selected curves are close to that of the actual curve. It is also seen that, for linear and exponential growth curves, the selected average service life is insensitive to the Transparent Band length in the range of 65% to 100% of the actual average service life. The stationary accounts show erratic behavior with respect to even the average service life (Figure 7.24). Again the indices are reasonably suggestive of the presence of an unfavorable parameter.

Right Modal Curves

The results are not very satisfactory in the case of right modal type curves also (Figures 6.25 to 6.27). Even the average service life becomes indeterminate till the

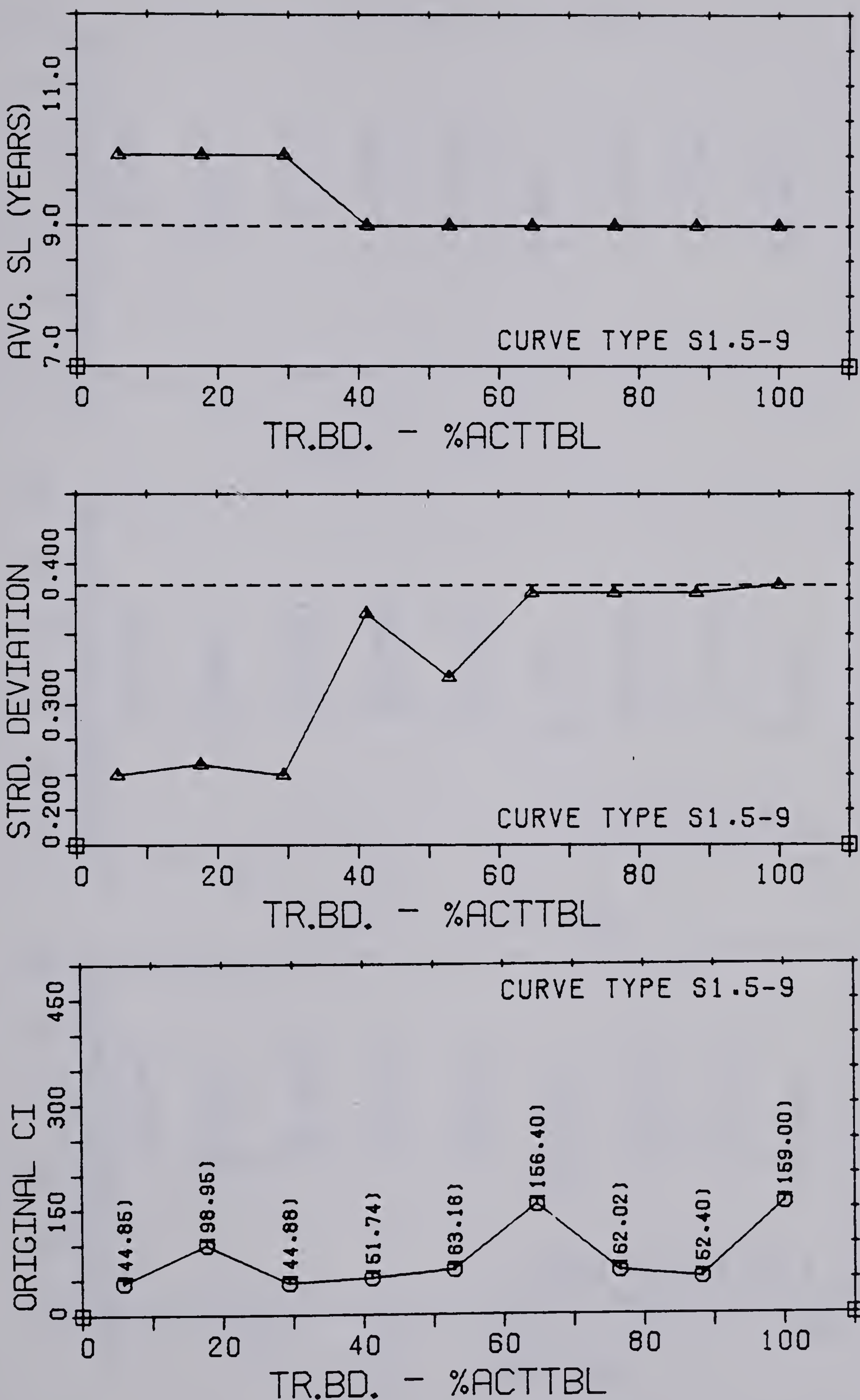


Figure 6.22 Results of the Investigation of the Transparent Band Length for a S(1.5)-9 Curve With a Linear Growth Rate of 3000 Units/Yr

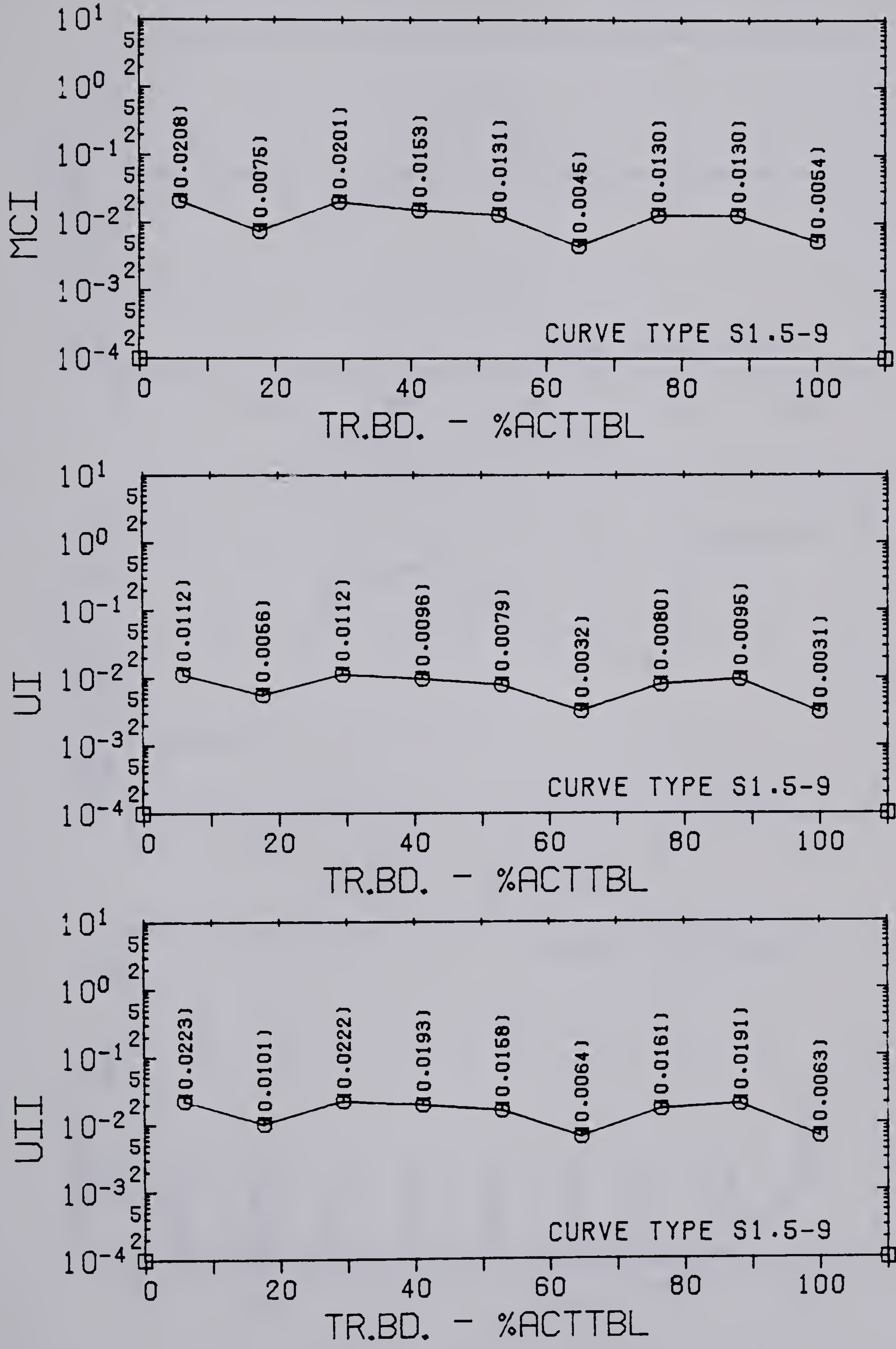


Figure 6.22 Continued from the Previous Page.

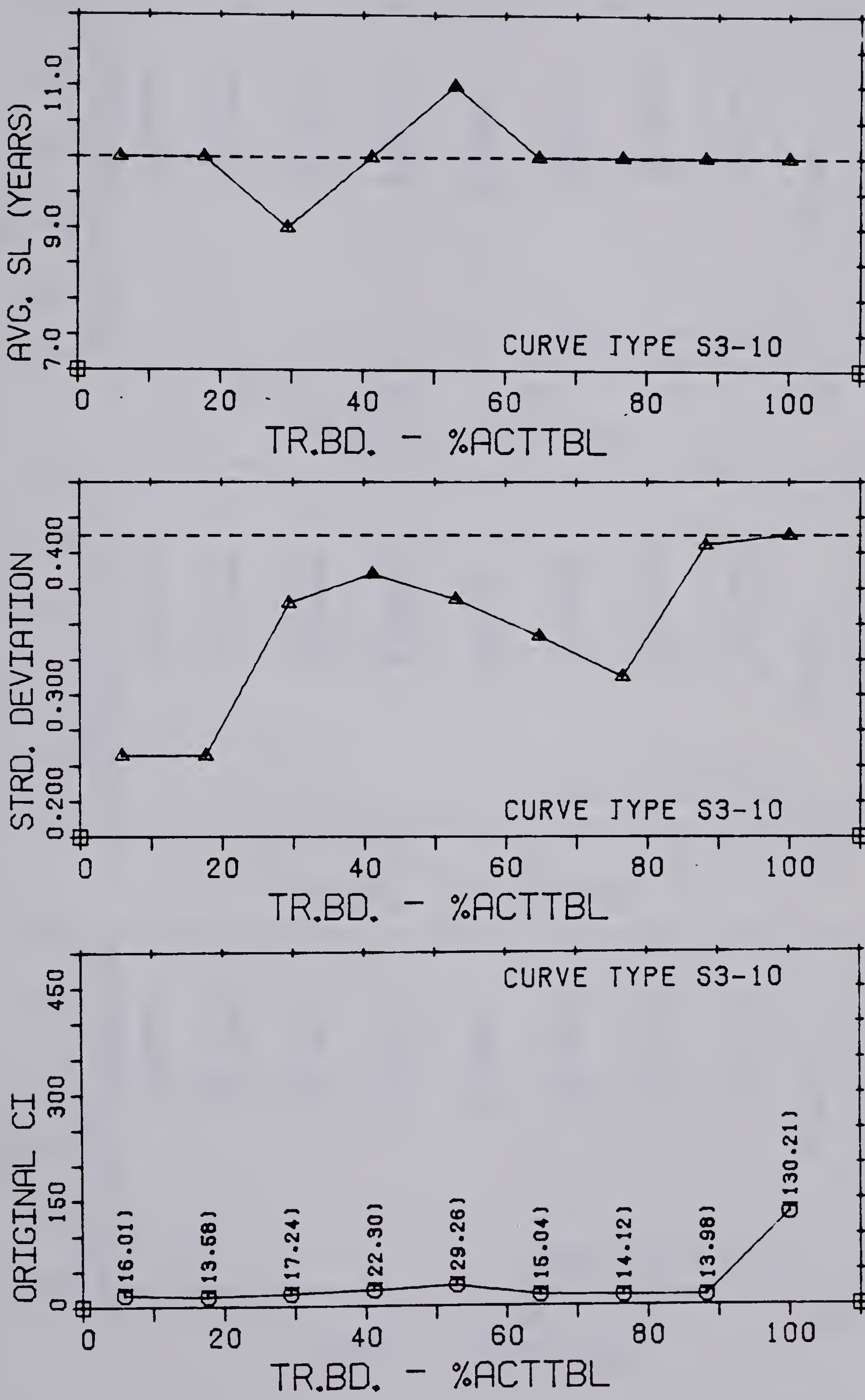


Figure 6.23 Results of the Investigation of the Transparent Band Length for a S3-10 Curve With an Exponential Growth Rate of 1.01

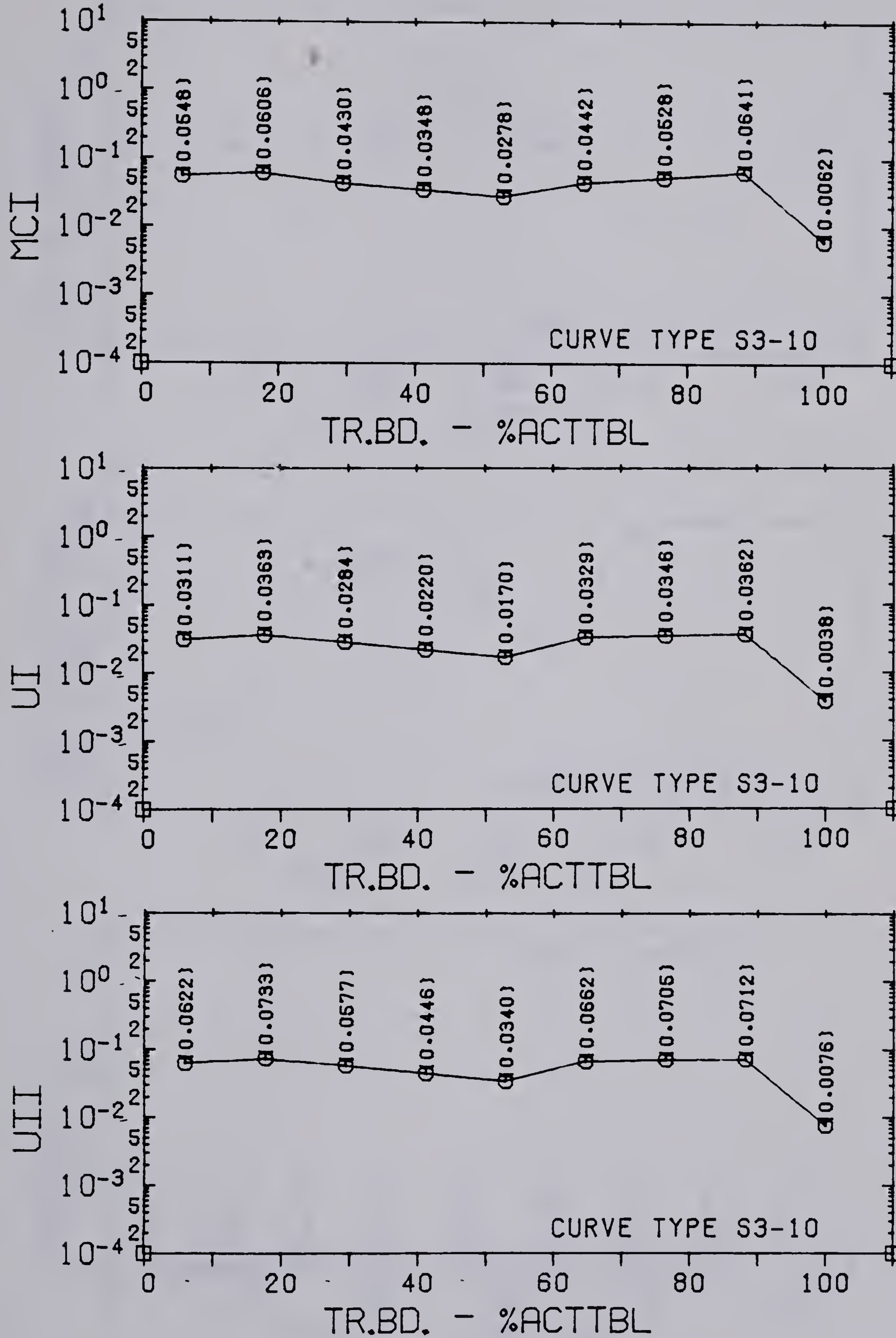


Figure 6.23 Continued from the Previous Page.

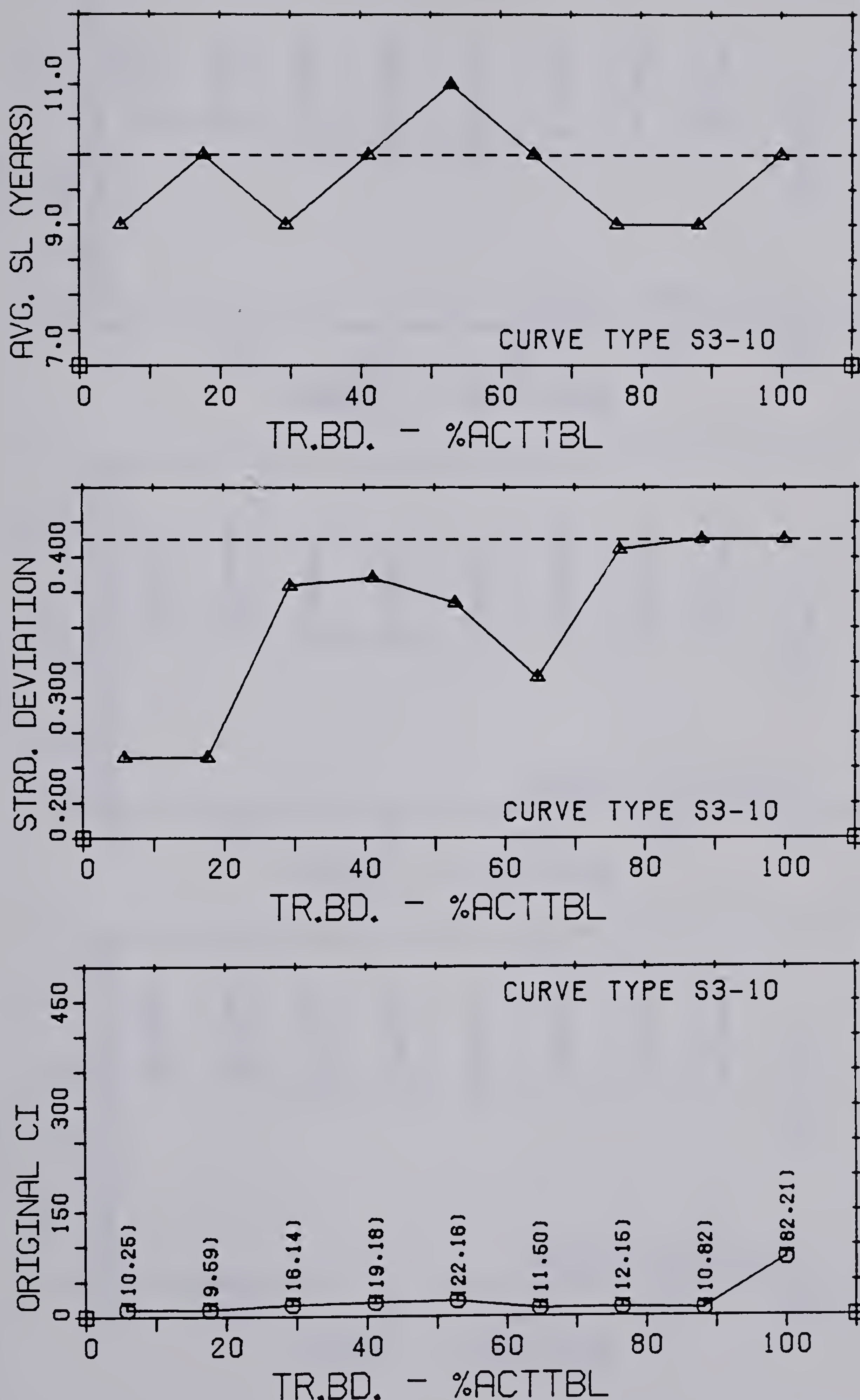


Figure 6.24 Results of the Investigation of the Transparent Band Length for a S3-10 Curve With a Stationary Plant Balance

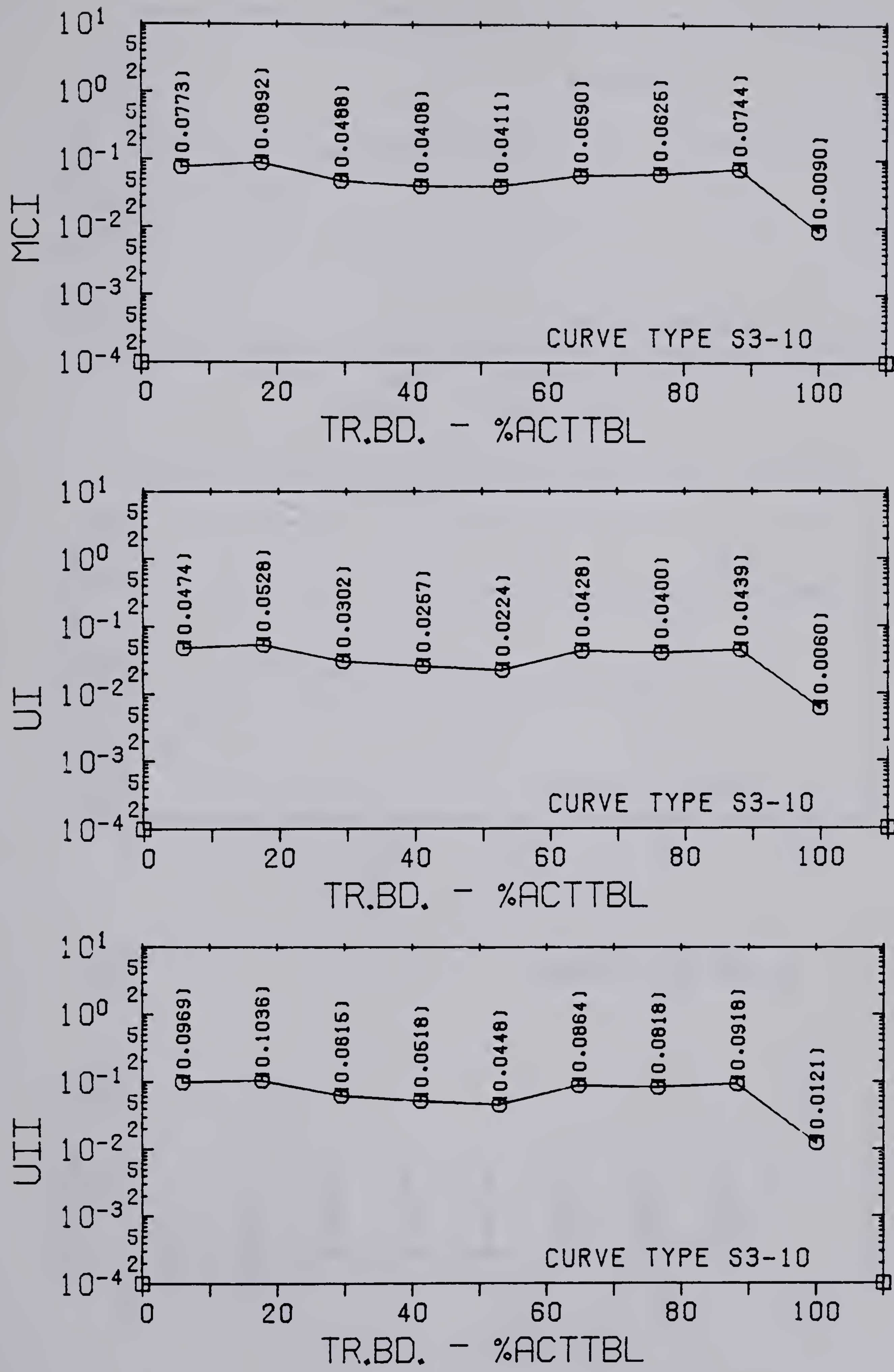


Figure 6.24 Continued from the Previous Page.

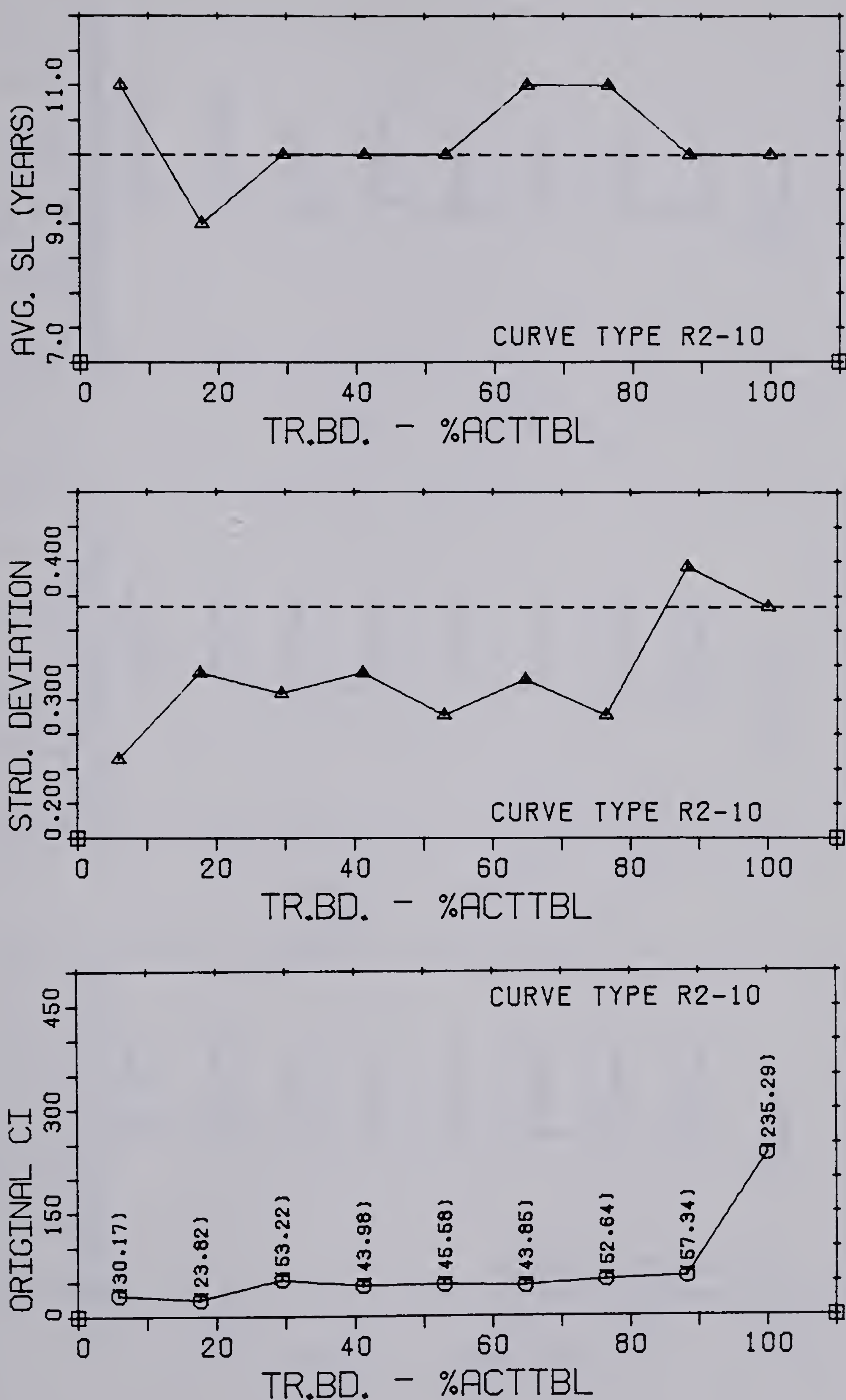
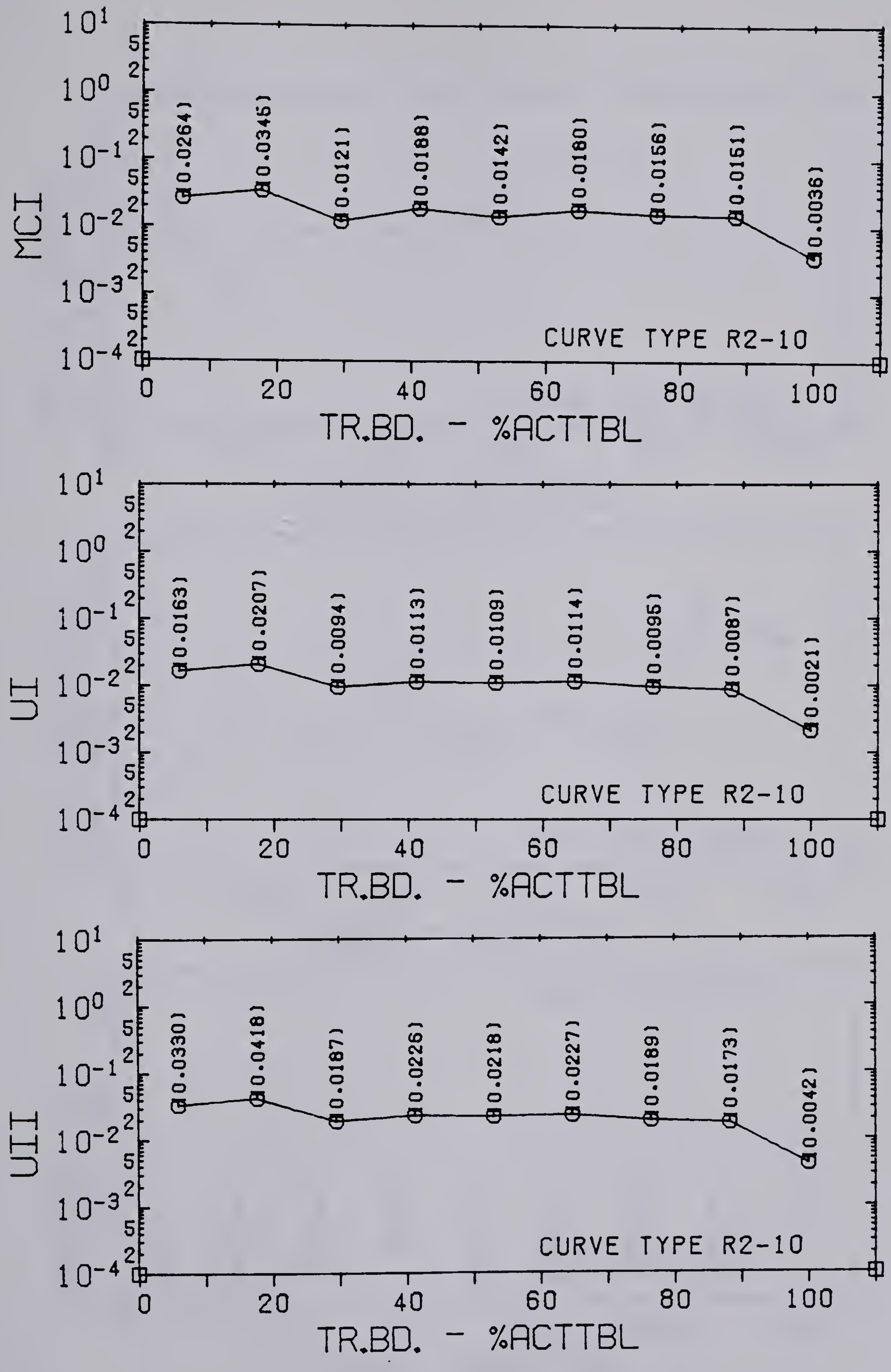


Figure 6.25 Results of the Investigation of the Transparent Band Length for a R2-10 Curve With a Linear Growth Rate of 2900 Units/Yr



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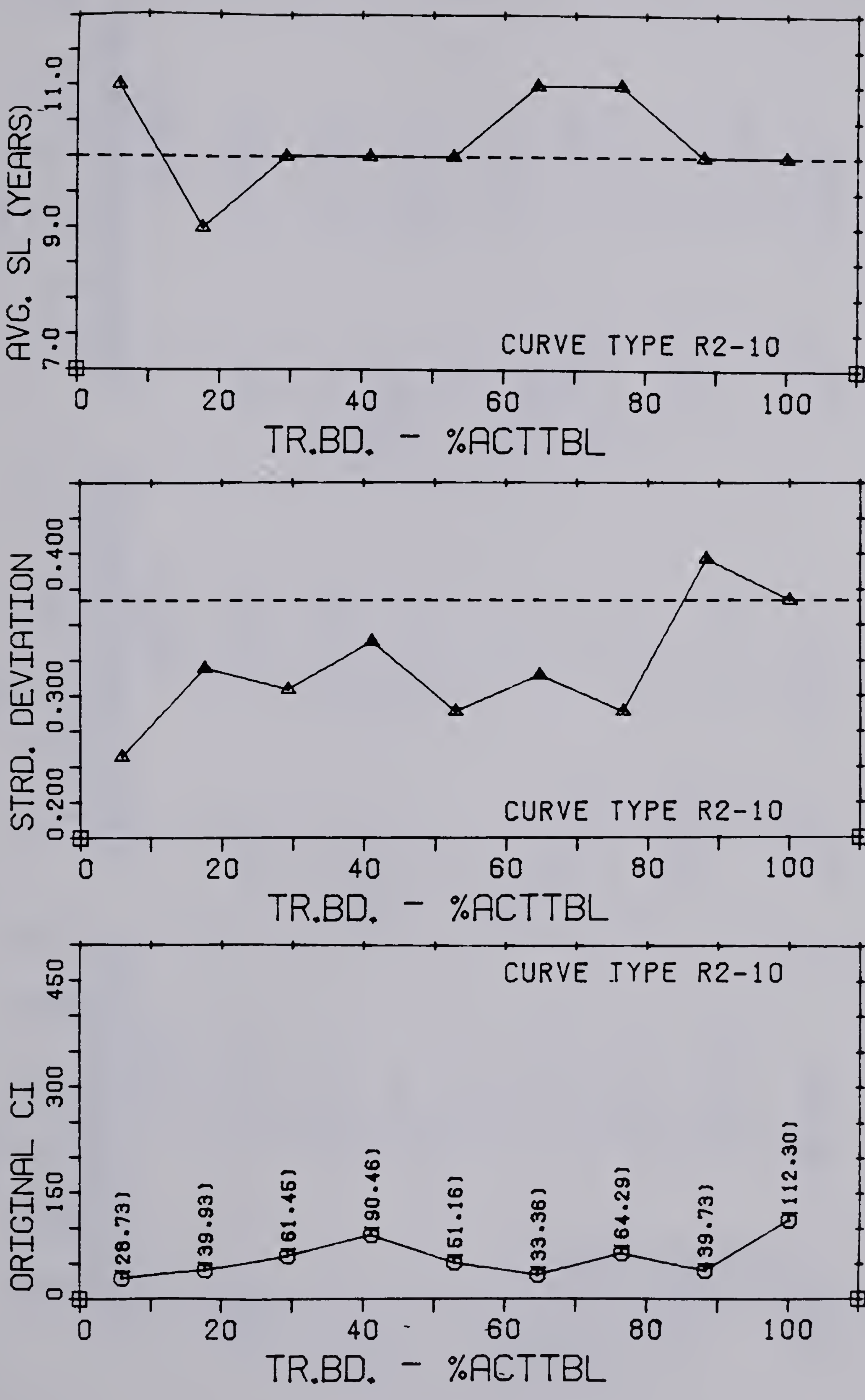

Figure 6.26 Results of the Investigation of the Transparent Band Length for a R2-10 Curve With an Exponential Growth Rate of 1.02

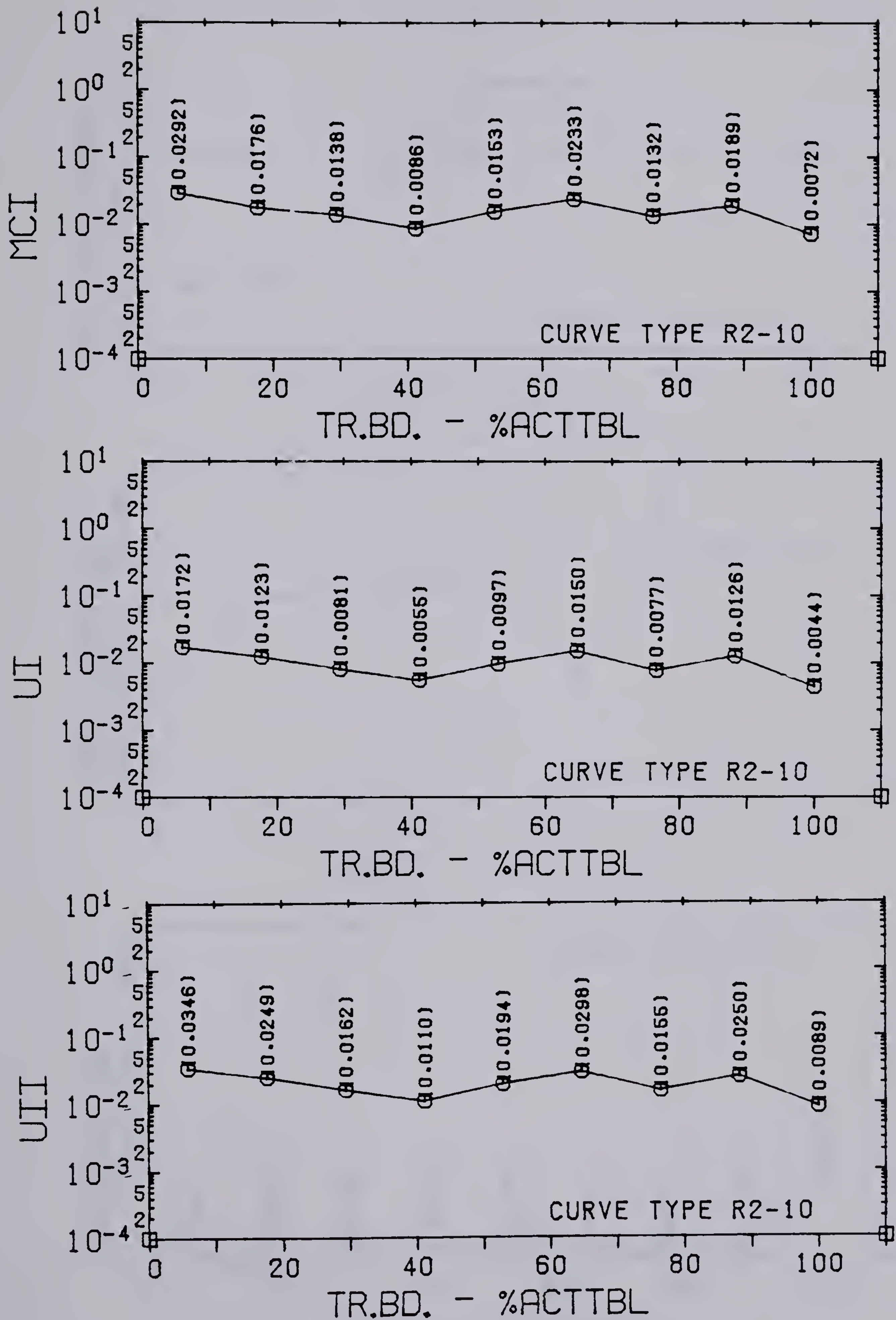

Figure 6.26 Continued from the Previous Page.

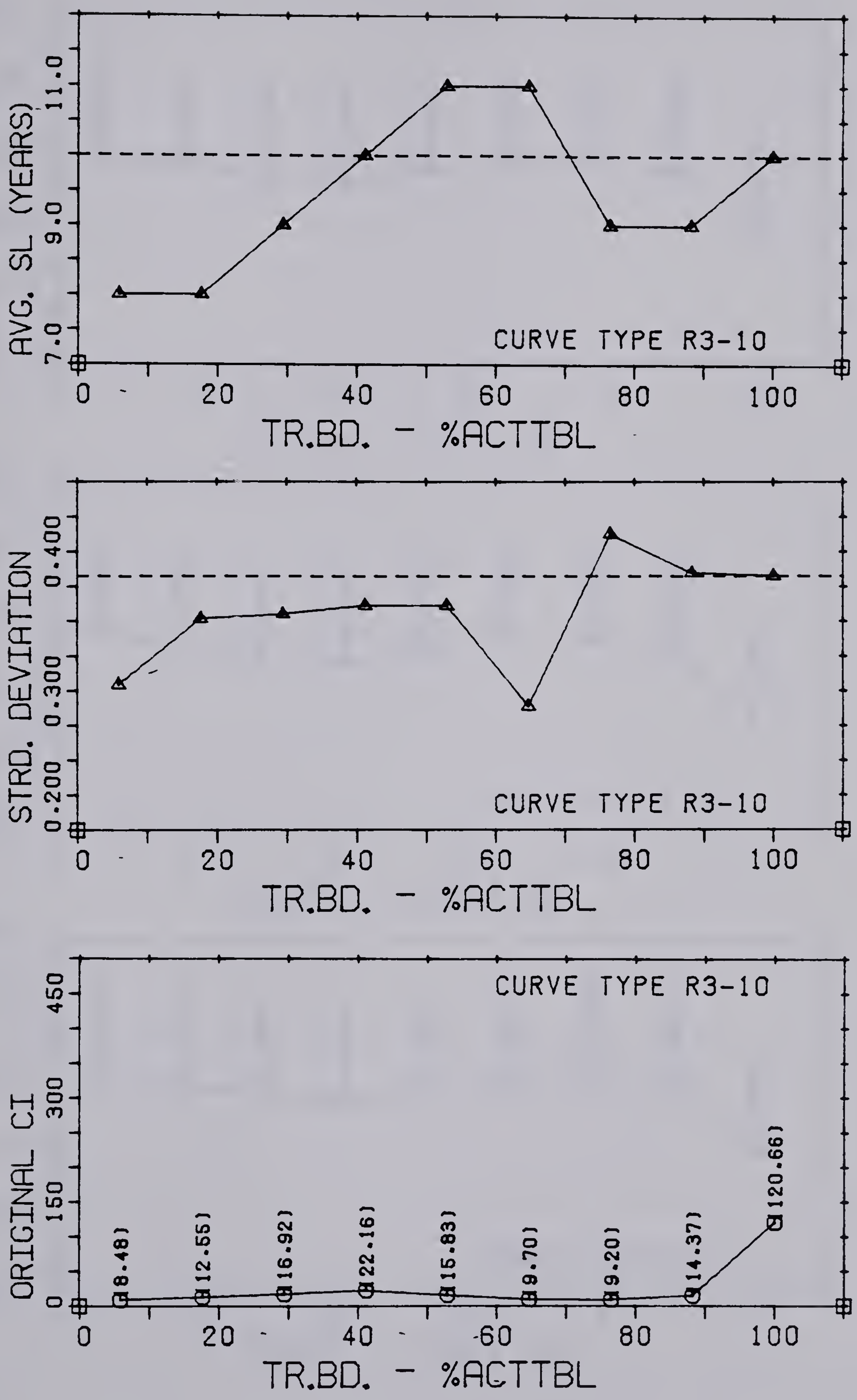

Figure 6.27 Results of the Investigation of the Transparent Band Length for a R3-10 Curve With a Stationary Plant Balance

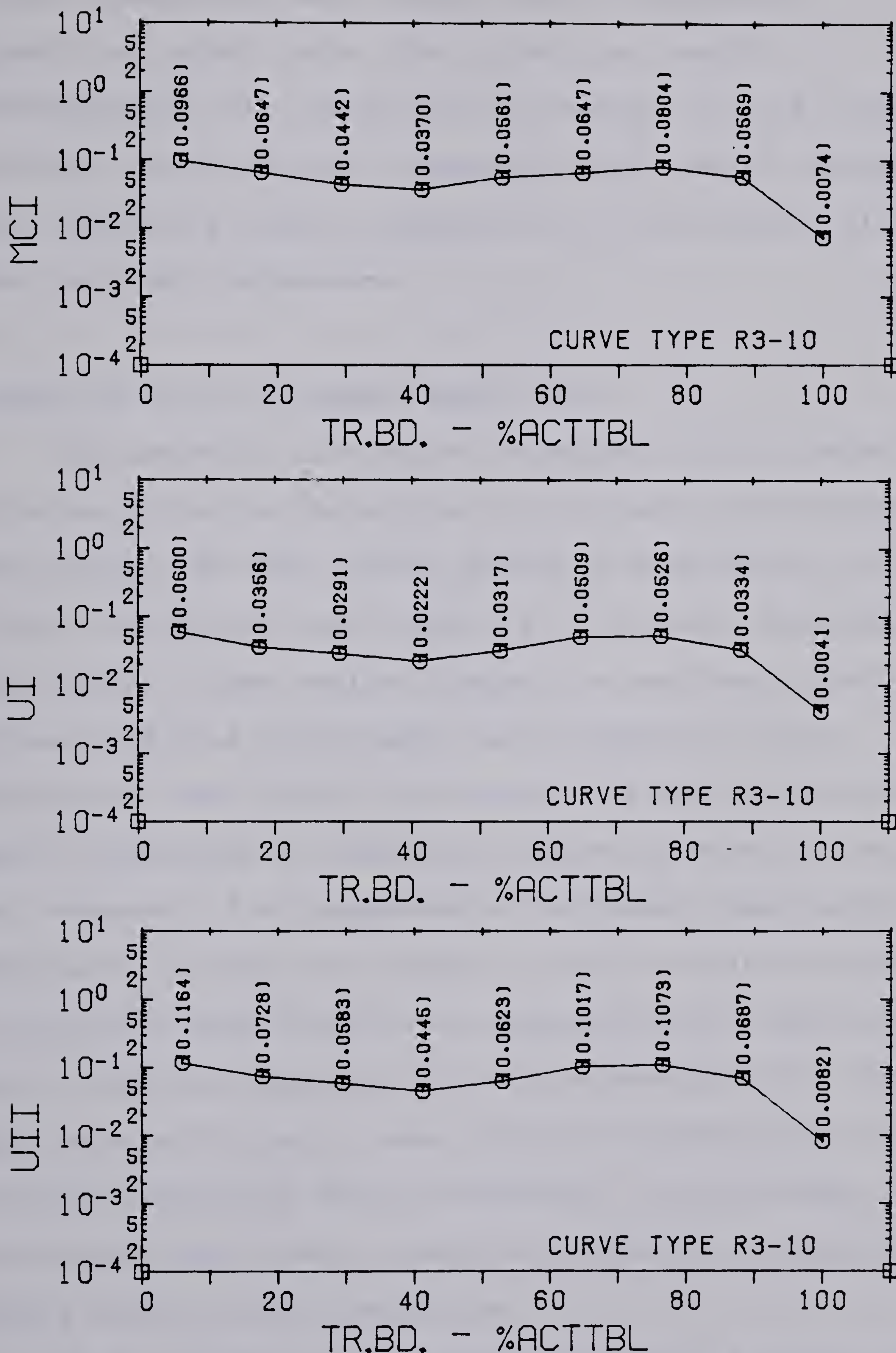

Figure 6.27 Continued from the Previous Page.

specified Transparent Band length is 90% or more of the actual Transparent Band length. As in the case of symmetrical modal curve, the curve type remains indeterminate till the specified Transparent Band length is identical to the actual Transparent Band length. However, the indices are clearly suggestive of the presence of one or more incorrect parameters.

Summary of the Transparent Band Tests

The preceding discussion indicates that the model is quite sensitive to the errors in the specified Transparent Band length. Extreme caution should be exercised if the actual Transparent Band length is not a known parameter. In particular, if the analyst suspects a stationary plant account and/or a right modal curve type and if the Transparent Band length is unknown, the method should not be used to determine the mortality characteristics of the data set. However, if an approximate Transparent Band length is known and if there are reasons to believe that the account is growing either linearly or exponentially, then the method can be used to determine the average service life. The curve type selected in such a case should be subjected to a critical evaluation before acceptance. In such cases, the Transparent Band length should be varied over a range of values before a final selection.

If the length of the Transparent Band is known, which usually will be the case, the method is applicable for the

determination of both the average service life and the curve type.

6.4 Growth Profile Tests

The sensitivity of the model to the specified growth profile type is subject to investigation in this section. The strategy employed for the test is somewhat the same as before: left modal, right modal and the symmetrical modal curves have been tested. For each curve type, six data sets have been used for the test. Of the six data sets simulated for each curve type, three are lower order and the other three are higher order curves; one each with linear, exponential and stationary profiles. The objective of this phase of the tests is to study the performance of the model if a growth profile other than the actual growth profile is erroneously specified by the analyst during the use of the model. The lengths of the Observation Band and the Transparent Band used for this phase of the study are those obtained from the previous two phases of the performance tests. The average service life used for the simulation of most of the data sets is 10 years; a few with 9 years and one set with 8 years. The data used for the tests have been simulated for 25 years each and the data for the last 8 years has been treated as the Observation Band data. Hence, the actual Transparent Band length for these data sets is 17 years which has been used for the tests (in accordance with the findings of the immediately preceding phase of the

tests). The specification of the optimum values of the Observation Band length and the Transparent Band length helps to reduce any distortions likely to be caused otherwise.

For each data set, all three growth profiles have been tested to determine the capability of the model to differentiate and select the correct mortality characteristics even if the specified growth profile is in error.

Tables 6.1 through 6.18 show the results of the tests. Of these 18 tables, the first six are for the data sets of linear actual growth profiles, the next six are for the exponential type and the last six are for stationary plant accounts. The actual characteristics of each data set (ie. the characteristics used in the Monte Carlo Simulator to simulate the data set) are as shown in the title of each table. The first row in all the tables is the specified growth profile identical to the actual growth profile of the data set being tested.

To aid the analyst in the specification of the growth profile and the range of growth rates over which the test has to be performed, a subroutine to fit a straight line to the Observation Band data has been provided in the model. This subroutine regresses a straight line to the actual plant balances in the Observation Band years, thereby giving an indication of the likely growth rate and profile of the plant account under investigation. If the growth profile is

either linear or exponential, the slope of the fitted straight line will be substantially higher than zero. Similarly, the slope of the fitted straight line will be quite close to zero, if the account is stationary. This feature of the model has been used to specify the ranges of the growth rates for all the test runs.

Linear Data Sets

Tables 6.1 to 6.6 show that for the data sets with an actual linear growth profile, the model successfully selects the correct characteristics when either the linear or the exponential growth profile is specified. The curve type selection is poor when the specified profile is a 'no-growth' type. Nevertheless, the indices are suggestive of the presence of a wrong parameter. However, the slope of the straight line fitted to the Observation Band (as discussed before) has been highly indicative of a growing account. This reduces the possibility of an erroneous specification of the growth profile. For all the cases, the correct average service life has been selected indicating its insensitivity to any errors in the growth profile specification.

Exponential Data Sets

Tables 6.7 to 6.12 show that the behavior of the exponential data sets is similar to that of the linear data sets. The selected characteristics have been accurate for

Table 6.1 Results of the Investigation of the Growth Profile for a L0-9 Curve With a Linear Growth Rate of 2000 Units/Yr.

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Linear	151.16	L0-09	0.0079	L0-09	0.0033	L0-09	0.0066	L0-09
Exponential	144.20	L0-09	0.0049	L0-09	0.0035	L0-09	0.0069	L0-09
No-Growth	44.92	L3-09	0.0159	L(0.5)-09	0.0108	L3-09	0.0213	L3-09

Table 6.2 Results of the Investigation of the Growth Profile for a L5-10 Curve With a Linear Growth Rate of 2500 Units/Yr

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Linear	159.40	L5-10	0.0049	L5-10	0.0031	L5-10	0.0061	L5-10
Exponential	32.78	L5-10	0.0273	L5-10	0.0150	L5-10	0.0298	L5-10
No-Growth	11.80	L4-10	0.0710	L4-10	0.0419	L4-10	0.0829	L4-10

Table 6.3 Results of the Investigation of the Growth Profile for a S(-0.5)-10 Curve With a Linear Growth Rate of 4000 Units/Yr

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Linear	148.96	S(-0.5)-10	0.0059	S(-0.5)-10	0.0034	S(-0.5)-10	0.0067	S(-0.5)-10
Exponential	76.79	R(0.5)-10	0.0102	S(-0.5)-10	0.0065	R(0.5)-10	0.0130	R(0.5)-10
No-Growth	23.62	S0-10	0.0297	S0-10	0.0212	S0-10	0.0423	S0-10

Table 6.4 Results of the Investigation of the Growth Profile for a S5-10 Curve With a Linear Growth Rate of 3000 Units/Yr

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Linear	571.74	S5-10	0.0015	S5-10	0.0008	S5-10	0.0017	S5-10
Exponential	85.42	S5-10	0.0092	S5-10	0.0056	S5-10	0.0112	S5-10
No-Growth	13.39	S4-10	0.0608	S4-10	0.0354	S4-10	0.0713	S4-10

Table 6.5 Results of the Investigation of the Growth Profile for a R(0.5)-10 Curve With a Linear Growth Rate of 3700 Units/Yr

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Linear	284.31	R(0.5)-10	0.0025	R(0.5)-10	0.0018	R(0.5)-10	0.0035	R(0.5)-10
Exponential	101.89	R(0.5)-10	0.0081	R(0.5)-10	0.0049	R(0.5)-10	0.0098	R(0.5)-10
No-Growth	20.96	S0-10	0.0317	S0-10	0.0238	S0-10	0.0477	S0-10

Table 6.6 Results of the Investigation of the Growth Profile for a R5-10 Curve With a Linear Growth Rate of 2500 Units/Yr

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Linear	134.81	R5-10	0.0056	R5-10	0.0036	R5-10	0.0071	R5-10
Exponential	33.60	R5-10	0.0270	R5-10	0.0144	R5-10	0.0286	R5-10
No-Growth	12.06	S5-10	0.0618	S5-10	0.0394	S5-10	0.0797	S5-10

Table 6.7 Results of the Investigation of the Growth Profile for a L0-10 Curve With an Exponential Growth Rate of 1.03

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Exponential	204.85	L0-10	0.0040	L0-10	0.0024	L0-10	0.0049	L0-10
Linear	223.81	L0-10	0.0036	L0-10	0.0022	L0-10	0.0045	L0-10
No-Growth	17.47	L1-10	0.0367	O5-10	0.0286	L1-10	0.0571	L1-10

Table 6.8 Results of the Investigation of the Growth Profile for a L4-10 Curve With an Exponential Growth Rate of 1.02

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Exponential	77.07	L4-10	0.0119	L4-10	0.0065	L4-10	0.0129	L4-10
Linear	68.06	L4-10	0.0125	L4-10	0.0073	L4-10	0.0146	L4-10
No-Growth	18.70	O1-11	0.0494	O1-11	0.0267	O1-11	0.0533	O1-11

Table 6.9 Results of the Investigation of the Growth Profile for a S(-0.5)-9 Curve With an Exponential Growth Rate of 1.04

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Exponential	156.14	S(-0.5)-09	0.0055	S(-0.5)-09	0.0032	S(-0.5)-09	0.0064	S(-0.5)-09
Linear	73.90	S(-0.5)-09	0.0115	S(-0.5)-09	0.0067	S(-0.5)-09	0.0135	S(-0.5)-09
No-Growth	20.74	S(-0.5)-09	0.0289	01-11	0.0240	S(-0.5)-09	0.0480	S(-0.5)-09

Table 6.10 Results of the Investigation of the Growth Profile for a S6-10 Curve With an Exponential Growth Rate of 1.03

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Exponential	252.91	S6-10	0.0033	S6-10	0.0018	S6-10	0.0035	S6-10
Linear	14.88	S6-10	0.0565	S6-10	0.0304	S6-10	0.0598	S6-10
No-Growth	5.45	S5-10	0.1502	S5-10	0.0790	S5-10	0.1635	S5-10

Table 6.11 Results of the Investigation of the Growth Profile for a R1-8 Curve With an Exponential Growth Rate of 1.03

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Exponential	245.01	R1-08	0.0034	R1-08	0.0020	R1-08	0.0041	R1-08
Linear	229.45	R1-08	0.0037	R1-08	0.0022	R1-08	0.0043	R1-08
NO-Growth	27.06	L1-08	0.0248	S(-0.5)-08	0.0184	L1-08	0.0368	L1-08

Table 6.12 Results of the Investigation of the Growth Profile for a R4-10 Curve With an Exponential Growth Rate of 1.03

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
Exponential	169.21	R4-10	0.0048	R4-10	0.0029	R4-10	0.0058	R4-10
Linear	58.48	R4-10	0.0135	R4-10	0.0085	R4-10	0.0169	R4-10
No-growth	14.52	S3-10	0.0570	R3-10	0.0341	S3-10	0.0681	S3-10

both the linear and exponential growth profile specifications. As before, the 'no-growth' type produces poor results. But, again, the slope of the regressed straight line to the Observation Band data has been clearly suggestive of a growing account and hence reduces the possibility of any errors in the growth profile specification. Tables 6.8 and 6.9 show some evidence of possible sensitivity of the selected average service life to errors in the specification of the growth profile, especially for a 'no-growth' type profile.

Stationary Data Sets

Tables 6.13 to 6.18 are for stationary data sets. These tables demonstrate the advantage of the procedure of regressing a straight line to the Observation Band data. The slope of the fitted straight line has been low for all the stationary data sets. This gives an indication of a stationary plant account and causes the analyst to assign very low growth rates even if he erroneously specifies linear or exponential growth profiles. This has produced excellent results for all the specified growth profiles. These tests indicate that the average service life is not very sensitive to the specified growth profile.

Summary of the Growth Profile Tests

The tests show that if the actual profile is either linear or exponential, the selected characteristics are

Table 6.13 Results of the Investigation of the Growth Profile for a L0-9 Curve With a Stationary Plant Account

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
No-Growth	149.39	L0-09	0.0057	L0-09	0.0033	L0-09	0.0067	L0-09
Linear	147.31	L0-09	0.0058	L0-09	0.0034	L0-09	0.0068	L0-09
Exponential	38.84	L0-09	0.0185	O5-09	0.0129	L0-09	0.0257	L0-09

Table 6.14 Results of the Investigation of the Growth Profile for a L5-10 Curve With a Stationary Plant Account

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
No-growth	151.34	L5-10	0.0043	L5-10	0.0031	L5-10	0.0062	L5-10
Linear	140.35	L5-10	0.0058	L5-10	0.0033	L5-10	0.0066	L5-10
Exponential	11.57	L5-10	0.0776	L5-10	0.0412	L5-10	0.0805	L5-10

Table 6.15 Results of the Investigation of the Growth Profile for a S(-0.5) Curve With a Stationary Plant Account

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
No-Growth	121.10	S(-0.5)-10	0.0068	S(-0.5)-10	0.0041	S(-0.5)-10	0.0082	S(-0.5)-10
Linear	121.03	S(-0.5)-10	0.0069	S(-0.5)-10	0.0041	S(-0.5)-10	0.0082	S(-0.5)-10
Exponential	34.88	S(-0.5)-10	0.0219	S(-0.5)-10	0.0143	S(-0.5)-10	0.0285	S(-0.5)-10

Table 6.16 Results of the Investigation of the Growth Profile for a S6-10 Curve With a Stationary Plant Account

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
No-Growth	102.29	S6-10	0.0070	S6-10	0.0035	S6-10	0.0069	S6-10
Linear	83.68	S6-10	0.0078	S6-10	0.0043	S6-10	0.0085	S6-10
Exponential	5.60	S6-10	0.1510	S6-10	0.0668	S6-10	0.1267	S6-10

Table 6.17 Results of the Investigation of the Growth Profile for a R1-10 Curve With a Stationary Plant Account

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
No-Growth	77.84	R1-10	0.0115	R1-10	0.0064	R1-10	0.0128	R1-10
Linear	97.62	R1-10	0.0092	R1-10	0.0051	R1-10	0.0102	R1-10
Exponential	66.40	R1-10	0.0111	R1-10	0.0075	R1-10	0.0150	R1-10

Table 6.18 Results of the Investigation of the Growth Profile for a R3-10 Curve With a Stationary Plant Account

	Conformance Index (CI)		Modified Conformance Index (MCI)		Theil's Index - 1 (UI)		Theil's Index - 2 (UII)	
Specified Growth Profile	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve	Value of the Index	Selected Curve
No-Growth	120.66	R3-10	0.0074	R3-10	0.0041	R3-10	0.0082	R3-10
Linear	113.94	R3-10	0.0078	R3-10	0.0043	R3-10	0.0087	R3-10
Exponential	21.37	R3-10	0.0389	R3-10	0.0233	R3-10	0.0462	R3-10

accurate when tested with either of the two specified profiles. However, if the data set is tested with a specified no-growth profile, the model does not select the right characteristics. Nevertheless, in such a case, the indices suggest the presence of one or more incorrect parameter(s).

In case of data sets with a stationary plant balance, low ranges of growth rates of linear and exponential types were assigned (because of the indication given by the slope of the straight line regressed to the Observation Band plant balances) when the linear and exponential profiles were specified for the test. The results obtained have been satisfactory for the no-growth profile for all the three types of specified profiles.

Thus, given the optimum lengths of the Observation Band and the Transparent Band, the model performs satisfactorily for all the growth profiles. If the growth profile is erroneous or unreasonable, the indices are highly suggestive of the presence of an incorrect parameter. However, the feature of regressing a straight line to the plant balances of the Observation Band reduces the likelihood of such a wrong specification of the growth profile.

6.5 Performance of the Indices

As mentioned earlier, the values of the indices and hence their performance has been recorded during the tests conducted in the first three phases of the tests on which

the following discussion is based.

All the indices are highly correlated which was expected due to the presence of the root mean squared value in three of the four indices. The performance of all the indices have been found to be identical except for the MCI (Modified Conformance Index which is the relative percent error) whose performance was found to be occasionally slightly different from that of others.

Although all the indices are identical with respect to performance, they can not be identical when it comes to specifying ranges of the values as a basis for grading the curves being tested.

For example, CI is a function of the Observation Band length. Hence it seems unreasonable to assign fixed ranges over which the curves selected could be graded as excellent, good, fair and bad. This aspect has already been discussed in detail elsewhere in the report.

The suggested grading scale for the Theil's Index - Type 1 is

0.000 to 0.003	Excellent
0.003 to 0.006	Good
0.006 to 0.009	Fair
0.009 to 1.000	Poor

This grading system is based on the values that have been recorded for the various tests (Figures 6.1 to 6.27 and Tables 6.1 to 6.18). A summary of these values of the index (UI) has been provided in Table 6.19. The UI values for the

Table 6.19 Summary of the Observations of the Theil's Index - Type 1 (UI)

Range	Both ASL & Curve Type Correct	Correct ASL Wrong Curve Type	Both ASL & Curve Type Wrong	Total
0.000 to 0.003	57	02	00	59
0.003 to 0.006	80	05	02	87
0.006 to 0.009	16	15	05	36
0.009 to 1.000	08	27	26	61
Total	161	49	33	243

Observation Band tests for which the Observation Band length was extremely small (less than 50% of the average service life) have not been considered in Table 6.19. The reasons for this exclusion will be discussed later in this section. A total of 243 cases were observed. Of these observations, 59 were in the range of 0 to 0.003, 87 were in the range of 0.003 to 0.006, 36 were in the range of 0.006 to 0.009 and 59 were in the range of 0.009 to 1.000.

It can be observed from Table 6.19 that out of a total of 59 cases in the range of 0 to 0.003, only two observations were with a wrong curve type and a correct average service life. There were no observations in this range with both the characteristics in error. This suggests that this range is difficult to achieve, sometimes even for the correct combinations of the characteristics because of the stochastic scatter. Hence this range has been graded as excellent. Similar arguments hold good for the remaining ranges specified in Table 6.19. These ranges have been graded as good, fair and poor for the reasons evident from the table.

A discussion on the behavior of the various indices for very low values of the Observation Band length follows.

As was expected (see discussion under section 4.1.2), when the available actual data is very small (short Observation Band lengths), exceptionally good values were obtained for all the indices even though the selected characteristics were unsatisfactory. This is because, if the

data for only one year is available, it is possible to closely approximate this value with many different combinations of even incorrect characteristics. However, with the increase in the available actual data, the number of combinations which can simulate the actual data set will be reduced till finally only the right combination or combinations close to the right combination will remain. Also, due to the requirement of matching larger number of stochastically distributed data points, the minimum achievable error may increase. This behavior is evident in most of the Figures 6.1 through 6.18.

This indicates that the values of the indices and hence the performance of the model becomes meaningless if the Observation Band length is excessively short (less than 50% of the average service life - refer to the section on Observation Band tests for more details). If this is in fact the case, the model should not be used for the estimation of the mortality characteristics even if the values of the indices appear to be favorable.

6.6 Applicability of the Model

1. The results obtained by the model are quite credible if the required amount of the Observation Band data is available in addition to a knowledge of the actual Transparent Band length.
2. As in any other method of life analysis, a favorable index (even with sufficient Observation Band length

etc.) should not be used as the sole criterion for selecting the mortality characteristics. The indices should be used only as qualitative guides in conjunction with other available information before reaching a conclusion about the life characteristics of any property. The estimates provided by the model is dependent only on the happenings of the past. The analyst should consider several other factors like the number of test years, generally known characteristics of the property being studied, past and forecasted economic conditions, changes in technology, changes in management policies etc. It will also be vital to incorporate the expertise and judgement of the analyst before arriving at a final estimate and forecast of the characteristics. It should be remembered that, as for any other model, the results from this model will be only a starting point and not the final values being sought.

7. SUMMARY AND CONCLUSIONS

7.1 Summary

The main objectives of this study are to provide an overview of the existing methods of life analysis and to suitably modify the existing Transparent Plant Balance Method of life analysis so as to overcome some of the inherent limitations of the method.

In compliance with the objectives, a brief overview of the various methods of life analysis has been provided. The Transparent Plant Balance Method has been discussed in greater detail than the others. This discussion covers the model, the logic involved and the adopted process of the calculations. Details of the study conducted by Tharumarajah [11] to evaluate the model has also been provided.

The performance of the Transparent Plant Balance Method (TPBM) and the probable reasons for the behavior have been discussed in detail. Based on this discussion, a modified version of the Transparent Plant Balance Method (MTPBM) has been proposed.

The MTPBM differs from the TPBM in its treatment of the plant balances. In the MTPBM, the plant balance has been treated as an independent variable of the system (in contrast to the TPBM wherein the plant addition is treated as an independent variable). The plant addition has been allowed to vary as a dependent variable of the plant balance (instead of treating the plant addition as an independent

variable as in the TPBM). As a result, the MTPBM extends the plant balances into the Transparent Band using a specified growth profile and growth rate. The plant additions for the Observation Band are simulated using the generated plant balances. This set of simulated plant additions is matched to the actual plant additions and the best combination of the parameters are selected. Three additional indices namely the Modified Conformance Index, Theil's Index - Type1 (UI) and Theil's Index - type2 (UII) [2,12] have been used to study their performance in conjunction with the model. Instead of using only an exponential growth profile, two additional profiles- linear and stationary- have been used.

A performance evaluation study was conducted wherein the sensitivity of the model to the variations in the Observation Band length, the Transparent Band length and the growth profiles were tested. Also, the behavior of the indices was studied during these tests.

A Monte Carlo Simulator was developed to generate the data required for the above mentioned performance evaluation study. The simulator generates the plant mortality data according to a specified growth profile, growth rate, average service life and Iowa Type Curve.

7.2 Conclusions

The following are the various conclusions of this study. The conclusions have been classified as:

1. conclusions related to the Observation Band,

2. conclusions related to the Transparent Band,
3. conclusions related to the growth profile, and
4. conclusions related to the general applicability of the model.

7.2.1 Conclusions Related to the Observation Band

1. For the left, symmetrical as well as the right modal curves, lower order curves require more actual data than the higher order curves for a satisfactory performance of the model.
2. The selected average service life is fairly insensitive to the Observation Band length. For the model to select the correct average service life, the Observation Band length should be 30% or more of the average service life.
3. The selected curve type is comparatively more sensitive to the Observation Band length. For the selection of the correct curve type, the Observation Band length should be at least 80% of the average service life.

   Hence for both the average service life and the curve type to be correct, the Observation Band length should be at least 80% of the average service life.

7.2.2 Conclusions Related to the Transparent Band

1. The model is very sensitive to the specified Transparent Band length. As such, the results should be treated with caution if the actual Transparent Band length is

unknown.

2. If the actual Transparent Band length is unknown and if there are reasons to believe that the account is stationary and/or conforms to a right modal curve type, this method of life analysis should not be used because there are strong indications that, for such a case, the mortality characteristics are indeterminate by this method.
3. If the actual Transparent Band length is known to be within a given range, and if there are reasons to believe that the growth rate is either linear or exponential, the method may be used to determine the average service life. Also, in such cases, the curve type selected should be used with caution and only after any necessary correction to compensate for any errors that might have been introduced. This limitation may be overcome to a certain extent if the tests are conducted with the Transparent Band length varied over the known range.

For example, if the actual Transparent Band length is known to be in the range of 8 to 11 years, the tests should be conducted with the Transparent Band length being incremented from 8 to 11 years in steps of one year each. Even then, the curve type selected should be treated with caution. The analyst should exercise his experience and expertise before accepting the selected curve type.

4. If the actual Transparent Band length is known, which usually will be the case, the model can be used to determine the average service life as well as the curve type.

7.2.3 Conclusions Related to the Growth Profile

Given the required Observation Band length and the Transparent Band length, the performance of the model is satisfactory for various growth profiles. However, if it is impossible to select the correct mortality characteristics with the specified growth profile, the various indices do a good job of clearly indicating such a situation.

7.2.4 Conclusions Related to the Indices

1. As expected, all the indices are highly correlated. However, to define a qualitative grading scale with which the selected curves may be ranked, Theil's index -Type 1 (UI) [2,12] is recommended because it has finite upper and lower boundaries. Also, unlike the Conformance Index (CI), it is not a function of the Observation Band length or any other input parameters of the model.
2. The suggested grading scale for UI is:

RANGE	GRADE
0.000 to 0.003	EXCELLENT
0.003 to 0.006	GOOD
0.006 to 0.009	FAIR

0.009 to 1.000	POOR

3. The above mentioned scale is not applicable (consequentially the model itself is not applicable) if the Observation Band length is small (0 to 80% of the average service life). Therefore, extreme caution is necessary in the use of the model and hence in the use of the specified ranges for the UI if the Observation Band length is smaller than that recommended (see section 7.2.1 for details).

7.2.5 Conclusions Related to the General Applicability of the Model

1. As is evident from the conclusions above, the results of the model are quite credible when provided with a favorable set of input parameters.
2. As with all the other existing models, the values of the indices should not be used as the sole criterion for the selection of the mortality characteristics. The result derived from the model is only a starting value. The analyst should incorporate his expertise and judgement in addition to considering several other factors like the magnitude of the indices, past and forecasted economic conditions, change in technology, changes in managerial policies etc.

7.3 Scope for Further Research

Before any such newly developed model could be implemented in real life, extensive research will have to be conducted to understand the behavior of the model(s) very thoroughly. The MTPBM is no exception to this requirement. Though the performance evaluation tests have been conducted during this study for the MTPBM, there is a wide scope for future research in this field.

1. In the present model, the plant additions are being matched in the Observation Band. This gives a good weightage to the recent trends in the growth of the plant balances which is a very desirable feature. However, if the actual growth is not as per any of the three growth profiles (ie. if the plant balances are yet to stabilize and are showing wide fluctuations - growth for a few years, stationary for a few more years, decline in between etc.), this method of matching might prove as a limitation. Under such circumstances, the model might yield better results if the retirements are matched rather than the plant additions. It would be interesting to study and compare the behavior of the model with both types of matching the actual and simulated data sets.
2. A statistical table specifying the values of the Theil's Index (UI) for different confidence limits could be developed.
3. The behavior of the model for data sets with trends in

the average service life and curve type is yet another aspect that may be studied.

4. In the present Monte Carlo Simulator, the data sets have been stochastic. But the specified growth rates have been fixed for the entire life of all the simulated data sets. The model could be tested with the data sets simulated having growth rates drawn each year from, perhaps, a normal distribution.
5. The Monte Carlo Simulator output has the complete aged data records for all the simulated data sets. These aged data sets could be analyzed by actuarial methods and the results could be compared with that of the model. This aspect will prove useful if the model is to be tested (as suggested in point three above) with data sets having trends in the average service life and the curve type.

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APPENDIX I

Variables Used in the MTPBM Computer Program

1. ACTASL - The actual average service life of the data set being tested.
2. ACTCUR - The actual curve with which the data has been simulated in the Monte Carlo Simulator.
3. ACTGRP - The actual growth profile of the data set being tested.
4. ACTSLP - The actual growth rate of the data set being tested.
5. ACTTBL - The actual Transparent Band length of the data set being tested.
6. APS - That part of the plant balance resulting from the plant additions of the previous years.
7. ASLI - Initial value of the specified range of the average service lives over which the test has to be conducted.
8. ASLF - Final Value of the specified range of the average service lives over which the test has to be conducted.
9. ASLS - The selected value of the average service life.
10. ASLT - The value of the average service life being used in the current iteration.
11. CURS - The curve selected by the MTPBM.
12. CURV - The type curve numbers of all the curves that have been specified for the test.
13. CURVT - The curve being tested in the current iteration.
14. ERE - Expected retirements in the current year from all

the vintages to date.

15. GA - The simulated gross plant additions.
16. GAA - Actual gross additions in the Observation Band.
17. GAACF - The gross additions in the Observation Band obtained by regressing a straight line to the actual gross additions.
18. IFLAG - Counters to decide the branching to be taken in the program.
19. NCURV - Total number of the standard type curves to be used to test the data sets.
20. OBS - The Transparent Band length of the selected curve.
21. PISA - Actual plant balance in the Observation Band.
22. PISACF - The plant balance in the Observation Band obtained by regressing a straight line to the actual plant balances.
23. PISG - The generated plant additions in the Transparent Band.
24. PSURV - Standard Iowa type survival tables.
25. SELS - Selected value of the indices.
26. SLOPE - Slope of the straight line fitted to the Observation Band data.
27. SLOPEI - Initial value of the range of the growth rates specified for the test.
28. SLOPF - Final value of the range of the growth rates specified for the test.
29. SLOPS - Growth rates of the selected curves.
30. SLPIN - The incremental value of the growth rate to be

used for the tests.

31. YA - Observation Band length

32. YNTCPT - 'Y' intercept of the straight line regressed to the Observation Band data.

33. YS - Transparent Band length

APPENDIX II

MTPBM Computer Program

```
      REAL PSURV(1395,10),GAA(20),PISA(20),ACTSLP
C
C THIS IS THE MAIN PROGRAM OF THE MTPBM. THIS SECTION OF THE
C PROGRAM ASSIGNES THE INPUT AND THE OUTPUT DEVICES, READS IN
C THE DATA AND CALLS THE SUBROUTINE DETRMN.
C
      INTEGER IFLAG(20),YA,YS,MAXYS,ASLI,ASLF,NCURV,CURV(31)
     +,ACTCUR,ACTASL,ACTGRP,ACTTBL
      COMMON/FLAG/IFLAG
      COMMON/CURV/CURV
      COMMON/P/PSURV
      COMMON/ASL/ASLI,ASLF
      COMMON/NC/NCURV
      COMMON/A/GAA,PISA
      COMMON/B/YA
      COMMON/G/YS
      COMMON/H/MAXYS
      COMMON/ACT/ACTCUR,ACTASL,ACTGRP,ACTSLP,ACTTBL
C
C ASSIGNS THE INPUT AND THE OUTPUT DEVICES
C
      CALL FTNCMD('ASSIGN 1=KRPR:CI(*L+1)',22)
      CALL FTNCMD('ASSIGN 2=KRPR:UI(*L+1)',22)
      CALL FTNCMD('ASSIGN 3=KRPR:ICURVE',20)
      CALL FTNCMD('DEFAULT 5=*SOURCE*',18)
      CALL FTNCMD('DEFAULT 6=*SINK*',16)
      CALL FTNCMD('ASSIGN 7=KRPR:OUTPUT(*L+1)',26)
      CALL FTNCMD('ASSIGN 8=KRPR:UII(*L+1)',23)
      WRITE(6,999)
999   FORMAT(//,'DO YOU WANT TO EMPTY KRPR:OUT?',/,' 0 - NO',
     +/,' 1 - YES')
      CALL FREAD(5,'I:',IEMP)
      IF(IEMP.NE.1)GOTO 995
      CALL CMD('$EMPTY KRPR:OUT Y',17)
995   CALL FTNCMD('ASSIGN 9=KRPR:OUT(*L+1)',23)
C
C READS IN ALL THE INPUT DATA.
C
      DO 90 J=1,1395
      READ(3,100)(PSURV(J,I),I=1,10)
90    CONTINUE
5     WRITE(9,3)
      WRITE(7,3)
3     FORMAT('1','****************************************')
      CALL FREAD(4,'I:',MNUM)
      WRITE(9,2)MNUM
      WRITE(6,2)MNUM
      WRITE(7,2)MNUM
2     FORMAT(///,'MODEL NUMBER=',I4)
      CALL FREAD(4,'I:',JJK)
      CALL FREAD(4,'I V:',IFLAG(1),JJK)
      CALL FREAD(4,'3I:',YA,YS,MAXYS)
      CALL FREAD(4,'R V:',GAA(1),YA)
      CALL FREAD(4,'R V:',PISA(1),YA)
```

```
      CALL FREAD(4,'2I:'.ASLI,ASLF)
      CALL FREAD(4,'1I:',NCURV)
      IF(NCURV.LT.31)GOTO 130
      DO 140 I=1,31
      CURV(I)=I
140   CONTINUE
      GOTO 150
130   CALL FREAD(4,'I V:',CURV(1),NCURV)
150   CALL FREAD(4,'4I:',ACTCUR,ACTASL.ACTGRP,ACTTBL)
      CALL FREAD(4,'1R:',ACTSLP)
70    CONTINUE
      WRITE(6,160)IFLAG(3),YS
160   FORMAT(/,'GR. PROFILE=',I2,5X,'TR. BAND=',I3)
C
C CALLS THE NEXT SUBROUTINE TO CONTINUE THE OPERATION.
C
      CALL DETRMN
      GOTO 120
100   FORMAT(10F11.6)
120   STOP
      END
C
C
C
C
      SUBROUTINE DETRMN
C
C THIS SUBROUTINE INITIALIZES ALL THE ARRAYS USED IN THE CALCULATIONS,
C CALLS THE SUBROUTINE TO REGRESS A STRAIGHT LINE TO THE OBSERVATION
C BAND DATA, CALLS THE APPROPRIATE SUBROUTINE TO EXTEND THE PLANT
C ADDITIONS INTO THE TRANSPARENT BAND AS PER A SPECIFIED GROWTH
C PROFILE, CALLS THE SUBROUTINE THAT SIMULATES THE PLANT ADDITIONS,
C AND FINALLY PRINTS ALL THE OUTPUT.
C
      REAL GAACF(20),PISACF(20),ACTSLP,ACCI1,ACCI2,ACREI,ACUI,
     +ACSLP,SEL(10,5,5),REIS(10,5),SLOPS(10,5),CI1(10),CI2(10),
     +CI3(10),CI4(10),GAA(20),PISA(20),CI5(10),C(10)
      INTEGER IFLAG(20),CURV(31),Y,ASLI,ASLF,ACASL,ACTES,ACTOB,
     +ACTCUR,ACTASL,ACTGRP,CURS(10,5),ASLS(10,5),OBS(10,5),YA
     +,IR1(10),IR2(10),IR3(10),IR4(10),P,CURVT,ASLT,IP,X,IR5(10)
     +,ACTTBL,YS
      COMMON/FLAG/IFLAG
      COMMON/SL/SLOPE,YNTCPT,GAACF,PISACF
      COMMON/SLP/SLOPEI,SLOPF,SLPIN
      COMMON/NC/NCURV
      COMMON/Y/Y,P
      COMMON/ASL/ASLI,ASLF
      COMMON/AT/ASLT
      COMMON/CURV/CURV
      COMMON/CUT/CURVT
      COMMON/ACDA/ACASL,ACTES,ACTOB,ACSLP,ACCI1,ACCI2,ACMCI1,
     +ACMCI2,ACREI,ACUI
      COMMON/ACT/ACTCUR,ACTASL,ACTGRP,ACTSLP,ACTTBL
      COMMON/OUT/CURS,ASLS,OBS,SLOPS,REIS,SEL
      COMMON/B/YA
      COMMON/A/GAA,PISA
      COMMON/IP/IP
      COMMON/H/MAXYS
      COMMON/MLM/MLM,MKM
      COMMON/G/YS
C
C CALLS THE SUBROUTINE TO REGRESS A STRAIGHT LINE TO OBSERVATION
C BAND DATA.
C
10    CALL STLINE
20    P=1
```

```
      Y=0
C
C INITIALIZES ALL THE VARIABLES.
C
      ACSLP=0.0
      ACCI1=0.0
      ACCI2=0.0
      ACMCI1=0.0
      ACMCI2=0.0
      ACREI=0.0
      ACASL=0
      ACTES=0
      ACTOB=0
      ACUI=0.0
       IJ=IFLAG(3)
      DO 760 I=1,10
      DO 170 J=1,5
      REIS(I,J)=0.0
      SLOPS(I,J)=0.0
      CURS(I,J)=0
      ASLS(I,J)=0
      OBS(I,J)=0
      DO 180 L=1,5
      SEL(I,J,L)=10000000.0
180   CONTINUE
170   CONTINUE
760   CONTINUE
      YS=YS-5
      MKM=0
C
C THE DO LOOP THAT ITERATES THE MTPBM FOR ALL THE COMBINATIONS
C OF THE INPUT PARAMETERS STARTS.
C
      DO 600 NNN=1,5
      MLM=0
      MKM=MKM+1
C
C SETS THE TRANSPARENT BAND LENGTH.
C
      YS=YS+5
      WRITE(6,620)YS
620   FORMAT(//,'TB LENGTH=',I3)
      DO 30 I=1,31
      ASLT=ASLI-1
C
C SETS THE CURVE TO BE USED FOR THE TESTS.
C
      CURVT=CURV(I)
      DO 140 L=1,20
C
C SETS THE ASL TO BE TESTED.
C
      ASLT=ASLT+1
      IF(I.NE.1)GOTO 5
      WRITE(6,6)ASLT
      WRITE(7,6)ASLT
6     FORMAT('ASLT=',I3)
5     IP=0
      DO 40 J=1,20
C
```

```
C CALLS THE SUBROUTINE TO EXTEND THE PLANT BALANCES INTO THE
C TRANSPARENT BAND.
C
      GOTO(60,60,70),IJ
60    CALL SLGRPR
      GOTO 90
 70   CALL PEGPR1
      GOTO 90
90    IF(IFLAG(2).EQ.1)GOTO 100
C
C CALLS THE SUBROUTINE THAT SIMULATES THE PLANT ADDITIONS.
C
 100  CALL GAAPIS
      GOTO 120
120   P=0
      IF(SLOPE.GT.SLOPF)GOTO 150
      IF(SLPIN.EQ.0)GOTO 150
40    CONTINUE
150   IF(ASLT.GE.ASLF)GOTO 160
140   CONTINUE
160   IF(I.GE.NCURV)GOTO130
30    CONTINUE
130   CONTINUE
      IF(YS.GE.MAXYS)GOTO 610
600   CONTINUE
610   CONTINUE
      Y=0
C
C OUTPUTS ALL THE CALCULATED VALUES
C
      WRITE(9,190)
      WRITE(7,190)
190   FORMAT(///,'OBSERVATION BAND DATA USED:',//,'YEAR',
     +3X,'PLANT IN SERVICE',3X,'GROSS ADDITIONS')
      DO 200 I=1,YA
      NX=1982-YA+I
      WRITE(9,210)NX,PISA(I),GAA(I)
      WRITE(7,210)NX,PISA(I),GAA(I)
210   FORMAT(' ',I4,6X,F10.0,7X,F9.0)
200   CONTINUE
      WRITE(9,220)ACTCUR,ACTASL,ACTGRP,ACTSLP,ACTTBL
      WRITE(7,220)ACTCUR,ACTASL,ACTGRP,ACTSLP,ACTTBL
220   FORMAT(///,'ACTUAL CURVE USED TO GENERATE THE DATA',
     +' ABOVE:',//,'CURVE=',I3,//,'ASL=',I3,//,'GROWTH PROFILE=',
     +I3,3X,'SLOPE=',F14.4,3X,'ACTUAL TB LENGTH=',I4)
      WRITE(9,230)ASLI,ASLF,SLOPEI,SLOPF,SLPIN
      WRITE(7,230)ASLI,ASLF,SLOPEI,SLOPF,SLPIN
230   FORMAT(///,'THE DATA USED IN TP BD:',//,'
     +CURVES TESTED: ALL 31 IOWA CURVES',//,'THE RANGE OF ASL',
     +' TESTED:',I3,' TO ',I3,' /INCREMENTS OF 1 YR.',//,
     +'GROWTH: LINEAR',//,'RANGE OF SLOPES TESTED:',F14.4,' TO ',
     +F14.4,' /INCREMENT OF',F14.4)
      WRITE(9,240)ACTES,ACASL,ACTOB,ACSLP,ACCI1,ACCI2,ACMCI1,
     +ACMCI2,ACREI,ACUI
      WRITE(7,240)ACTES,ACASL,ACTOB,ACSLP,ACCI1,ACCI2,ACMCI1,
     +ACMCI2,ACREI,ACUI
240   FORMAT(///,'THE FOLLOWING ARE THE VARIOUS CALCULATED',
     +' VALUES FOR THE ACTUAL CURVE:',//,'CURVE=',I3,3X,'ASL=',
     +I3,3X,'OBS. BAND=',I3,3X,'SLOPE=',F14.4,//,'CI 1=',F10.2,
     +3X,'CI 2=',F10.2,3X,'MCI 1=',F8.5,3X,'MCI 2=',F8.5,3X,'REI=',F8.5,
```

```
     +'UI= ',F9.6)
      DO 260 I=1,10
      IR1(I)=I
      IR2(I)=I
      IR3(I)=I
      IR4(I)=I
      IR5(I)=I
      CI1(I)=SEL(I,1,1)
      CI2(I)=SEL(I,2,2)
      CI3(I)=SEL(I,3,3)
      CI4(I)=SEL(I,4,4)
      CI5(I)=SEL(I,5,5)
  260 CONTINUE
      CALL VSRTR(CI1,10,IR1)
      CALL VSRTR(CI2,10,IR2)
      CALL VSRTR(CI3,10,IR3)
      CALL VSRTR(CI4,10,IR4)
      CALL VSRTR(CI5,10,IR5)
      DO 440 L=1,5
      GOTO(400,410,420,430,500),L
  400 WRITE(9,270)IFLAG(3)
      WRITE(7,270)IFLAG(3)
  270 FORMAT(///,'THE FOLLOWING CURVES HAVE BEEN SELECTED',
     +' BASED ON THE ORIGINAL CI - CI1:',//,'(THE GROWTH PROFILE',
     +' USED IS - ',I2)
  450 WRITE(9,280)
      WRITE(7,280)
  280 FORMAT(//,'CURVE#',3X,'TRS. BD. LNTH.',3X,'ASL',3X,'SLP. OF LN.'
     +,8X,'CI1',10X,'UI1',12X,'MCI1',10X,'MCI2',8X,'UI',11X,'REI',/)
      GOTO 460
  410 WRITE(9,310)IFLAG(3)
      WRITE(7,310)IFLAG(3)
  310 FORMAT(///,'THE FOLLOWING CURVES HAVE BEEN SELECTED BASED',
     +' ON THEIL''S INDEX - UI1:',//,'(THE GROWTH ',
     +'PROFILE USED IS - ',I2)
      GOTO 450
  420 WRITE(9,330)IFLAG(3)
      WRITE(7,330)IFLAG(3)
  330 FORMAT(///,'THE FOLLOWING CURVES HAVE BEEN SELECTED BASED',
     +' ON MODIFIED CI - MCI1:',//,'(THE GROWTH PROFILE USED IS - ',
     +I2)
      GOTO 450
  430 WRITE(9,340)IFLAG(3)
      WRITE(7,340)IFLAG(3)
  340 FORMAT(///,'THE FOLLOWING CURVES HAVE BEEN SELECTED BASED',
     +' ON CURVE FITTED MCI - MCI2',I2)
      GOTO 450
  500 WRITE(9,510)IFLAG(3)
      WRITE(7,510)IFLAG(3)
  510 FORMAT(///,'THE FOLLOWING CURVES HAVE BEEN SELECTED BASED',
     +' ON THEIL''S FORECAST COEFFICIENT:',//,'(THE GROWTH',
     +'PROFILE USED',
     +' IS - ',I2,')')
      GOTO 450
  460 DO 290 I=1,10
      IF(L.EQ.1)N=IR1(I)
      IF(L.EQ.2)N=IR2(I)
      IF(L.EQ.3)N=IR3(I)
      IF(L.EQ.4)N=IR4(I)
      IF(L.EQ.5)N=IR5(I)
```

```
      IF(SEL(N,1,L).EQ.0.)SEL(N,1,L)=.001
      IF(SEL(N,2,L).EQ.0.)SEL(N,2,L)=.001
      SEL1=1./SEL(N,1,L)
      SEL2=SEL(N,2,L)
      WRITE(9,300)CURS(N,L),OBS(N,L),ASLS(N,L),SLOPS(N,L),SEL1,SEL2,
     +SEL(N,3,L),SEL(N,4,L),SEL(N,5,L),REIS(N,L)
      WRITE(7,300)CURS(N,L),OBS(N,L),ASLS(N,L),SLOPS(N,L),SEL1,SEL2,
     +SEL(N,3,L),SEL(N,4,L),SEL(N,5,L),REIS(N,L)
300   FORMAT(2X,I2,11X,I2,10X,I2,F14.4,1X,F12.2,1X,F14.6,1X,F13.4,
     +1X,F13.4,3X,F9.6,5X,F9.5)
290   CONTINUE
440   CONTINUE
      DO 820 I=1,10
      C(I)=1./SEL(I,1,1)
820   CONTINUE
      WRITE(1,800)(C(I),I=1,10)
      WRITE(8,810)(SEL(I,2,2),I=1,10)
      WRITE(2,810)(SEL(I,5,5),I=1,10)
800   FORMAT(10F12.2)
810   FORMAT(10F12.7)
      RETURN
      END
C
C
C
C
      SUBROUTINE STLINE
C
C THIS SUBROUTINE REGRESSES A STRAIGHT LINE TO THE OBSERVATION
C BAND DATA.
C
      REAL SLOPE,YNTCPT,GAA(20),PISA(20),GAACF(20),PISACF(20),X(20),
     +Y(20),SUMX,SUMXS,SUMY,SUMYS,SUMXY,N,YCF(20),A,B,C
      INTEGER IFLAG(20),YA,Z,ZZ
      COMMON/FLAG/IFLAG
      COMMON/SL/SLOPE,YNTCPT,GAACF,PISACF
      COMMON/A/GAA,PISA
      COMMON/B/YA
      Z=0
      ZZ=0
10    DO 30 I=1,20
      Y(I)=GAA(I)
      X(I)=I-1
      IF(I.GE.YA)GOTO 200
30    CONTINUE
200   IF(IFLAG(2).EQ.1)ZZ=1
      GOTO 40
20    DO 50 I=1,20
      Y(I)=PISA(I)
      X(I)=I-1
      IF(I.GE.YA)GOTO 210
50    CONTINUE
210   IF(IFLAG(2).NE.1)ZZ=1
40    SUMX=0.0
      SUMXS=0.0
      SUMY=0.0
      SUMYS=0.0
      SUMXY=0.0
      N=YA
      DO 60 I=1,20
```

```
        SUMX=SUMX+X(I)
        SUMXS=SUMXS+(X(I))**2
        SUMY=SUMY+Y(I)
        SUMYS=SUMYS+(Y(I))**2
        SUMXY=SUMXY+(X(I)*Y(I))
        IF(I.GE.YA)GOTO 70
 60     CONTINUE
 70     YNTCPT=((SUMXS*SUMY)-(SUMX*SUMXY))/((N*SUMXS)
       +-(SUMX)**2)
        A=((N*SUMXY)-(SUMX*SUMY))
        SLOPE=A/((N*SUMXS)-(SUMX)**2)
        IF(ZZ.NE.1)GOTO 140
        ZZ=0
  170   WRITE(6,130)SLOPE,YNTCPT
        WRITE(9,130)SLOPE,YNTCPT
        WRITE(7,130)SLOPE,YNTCPT
  130   FORMAT(///,'The following straight line has been fitted',
       +' TO THE OBSERVATION',//,'BAND DATA:',//,'SLOPE=',F16.5,
       +//,'  Y INTERCEPT=',F16.5)
 140    IF(Z.NE.0)GOTO 90
 80     DO 100 I=1,20
        YCF(I)=YNTCPT+(SLOPE*X(I))
        GAACF(I)=FLOAT(IFIX(YCF(I)+.5))
        IF(I.GE.YA) GOTO 110
 100    CONTINUE
        GOTO 110
 90     DO 120 I=1,20
        YCF(I)=YNTCPT+(SLOPE*X(I))
        PISACF(I)=FLOAT(IFIX(YCF(I)+.5))
        IF(I.GE.YA)GOTO 110
 120    CONTINUE
 110    IF(Z.NE.0)GOTO 190
        Z=1
        GOTO 20
 190    CONTINUE
 240    CONTINUE
        RETURN
        END
C
C
C
C
        SUBROUTINE SLGRPR
C
C THIS SUBROUTINE EXTENDS THE PLANT BALANCES INTO THE TRANSPARENT
C BAND USING EITHER A STATIONARY PROFILE OR A LINEAR PROFILE (WITH
C POSITIVE SLOPE.
C
        REAL GAA(20),PISA(20),SLOPE,YNTCPT,GAACF(20),PISACF(20),
       +GVALUE(100),GAG(100),PISG(100),SLOPEI,SLOPF,SLPIN
        INTEGER IFLAG(20),YS1,YS,Y,II
        COMMON/FLAG/IFLAG
        COMMON/A/GAA,PISA
        COMMON/SL/SLOPE,YNTCPT,GAACF,PISACF
        COMMON/B/YA
        COMMON/G/YS
         COMMON/II/II
        COMMON/Y/Y,P
        COMMON/GEN/GAG,PISG,YS1
        COMMON/SLP/SLOPEI,SLOPF,SLPIN
```

```
      COMMON/IP/IP
      IF(Y.EQ.1)GOTO 120
      IF(IFLAG(3).NE.1)GOTO 280
      SLOPEI=0.
      SLOPF=0.
      SLPIN=0.
      WRITE(7,300)
      WRITE(9,300)
300   FORMAT(//,'GROWTH PROFILE USED FOR TESTING IS 1')
      GOTO 261
280   WRITE(6,260)
260   FORMAT(///,'ENTER INITIAL SLOPE,FINAL SLOPE & INCREMENTAL SLOPE')
      CALL FREAD(5,'3R:',SLOPEI,SLOPF,SLPIN)
      WRITE(7,310)
      WRITE(9,310)
310   FORMAT(//,'GROWTH PROFILE USED FOR TESTING IS 2')
      IF(SLOPEI.LE.YNTCPT)GOTO 261
      WRITE(6,270)YNTCPT
270   FORMAT(///,'**ERROR ENCOUNTERED**',//,'THE INITIAL',
     +' SLOPE SHOULD NOT BE MORE THAN',F10.0,//,'TRY AGAIN')
      GOTO 280
261   SLOPE=SLOPEI
120   IF(IP.EQ.0)SLOPE=SLOPEI
      IF(IFLAG(2).NE.1)GOTO 150
140   IF(IFLAG(4).EQ.1)YNTCPT=GAACF(1)
      IF(IFLAG(4).EQ.2)YNTCPT=GAA(1)
      GOTO 160
150   IF(IFLAG(4).EQ.1)YNTCPT=PISACF(1)
      IF(IFLAG(4).EQ.2)YNTCPT=PISA(1)
      GOTO 160
160   IF(IFLAG(7).NE.1)GOTO 100
170   SLOPEI=(YS)/(YNTCPT-0.0)
      SLOPF=SLOPEI
      SLPIN=0.0
      SLOPE=SLOPEI
100   II=0
      DO 180 I=1,YS
      GVALUE(I)=YNTCPT+(SLOPE*(0.0-FLOAT(I)))
      J=I
      IF(GVALUE(I).LT.0)GOTO 190
180   CONTINUE
      GOTO 200
190   YS1=J-1
      II=1
200   IF(II.EQ.1)N=YS1
      IF(II.NE.1)N=YS
      K=N
      IF(IFLAG(2).NE.1)GOTO 220
210   DO 230 I=1,N
      GAG(K)=FLOAT(IFIX(GVALUE(I)+.5))
      PISG(K)=0.0
      K=K-1
230   CONTINUE
      GOTO 240
220   DO 250 I=1,N
      PISG(K)=FLOAT(IFIX(GVALUE(I)+.5))
      GAG(K)=0.0
      K=K-1
250   CONTINUE
240   Y=1
```

```
      IP=1
      RETURN
      END
C
C
C
C
      SUBROUTINE GAAPIS
C
C THIS SUBROUTINE SIMULATES THE PLANT ADDITIONS USING THE PLANT
C BALANCES GENERATED BY ONE OF THE GROWTH PROFILE SUBROUTINES.
C
      REAL GAA(20),PISA(20),GAG(100),PISG(100),GA(120),APS(120),
     +PI(120),GAACF(20),PISACF(20),SLOPS(10,5),ACSLP,ACREI,ACUI,
     +PSURV(1395,10),ACTSLP,CIC(5),REIS(10,5),SEL(10,5,5),ERE(120)
      INTEGER IFLAG(20),Y,P,ROW,COLMN,ACTCUR,ACTASL,ACTGRP,ASLT
     +,ACASL,ACTES,ACTOB,CURS(10,5),ASLS(10,5),OBS(10,5),YA,YS,YS1
     +,ASLI,ASLF,II,CURVT,X,XX,ACTTBL
      COMMON/FLAG/IFLAG
      COMMON/A/GAA,PISA
      COMMON/B/YA
      COMMON/G/YS
      COMMON/GEN/GAG,PISG,YS1
      COMMON/II/II
      COMMON/Y/Y,P
      COMMON/CUT/CURVT
      COMMON/ASL/ASLI,ASLF
      COMMON/SL/SLOPE,YNTCPT,GAACF,PISACF
      COMMON/P/PSURV
      COMMON/ACT/ACTCUR,ACTASL,ACTGRP,ACTSLP,ACTTBL
      COMMON/ACDA/ACASL,ACTES,ACTOB,ACSLP,ACCI1,ACCI2,ACMCI1,ACMCI2,
     +ACREI,ACUI
      COMMON/OUT/CURS,ASLS,OBS,SLOPS,REIS,SEL
      COMMON/AT/ASLT
      COMMON/SLP/SLOPEI,SLOPF,SLPIN
      COMMON/H/MAXYS
      COMMON/MLM/MLM,MKM
      IF(P.GT.1)GOTO 5
      DO 15 I=1,120
      APS(I)=0.0
      ERE(I)=0.
   15 CONTINUE
    5 IF(II.EQ.1)N=YS1+1
      IF(II.EQ.0)N=YS+1
      IF(P.NE.1)GOTO 340
      WRITE(6,280)N
  280 FORMAT('N1=',I3)
  340 J=N-1
      DO 10 I=1,J
      PI(I)=FLOAT(IFIX(PISG(I)+.5))
   10 CONTINUE
      JX=J+YA
      DO 20 I=N,120
      PI(I)=PISA(I-J)
      IF(I.GE.JX)GOTO 30
   20 CONTINUE
C
C CALCULATES THE PERCENT SURVIVING AT THE END OF THE FIRST YEAR
C FOR THE PROPOSED NEW PLANT ADDITIONS.
C
```

```
      30    KHALF=IFIX(100.*(0.5/FLOAT(ASLT))+0.5)
            IF(KAHALF.EQ.0)GOTO 140
            NN=KHALF-(KHALF/10)*10
            IF(NN)120,130,120
      130   ROW=IFIX(KHALF/10.)+(CURVT-1)*45
            COLMN=10
            GOTO 140
      120   ROW=(CURVT-1)*45+(KHALF/10)+1
            COLMN=KHALF-(KHALF/10)*10
      140   IF(KHALF.EQ.0)PSUR=1.
            IF(KHALF.NE.0)PSUR=PSURV(ROW,COLMN)
            DO 40 L=1,120
      C
      C CALCULATES THE PLANT ADDITIONS FOR THE CURRENT YEAR
      C
            GA(L)=FLOAT(IFIX(((PI(L)-APS(L)+ERE(L))/PSUR)+.5))
            FIRST=GA(L)
            K=L
            DO 70 I=1,120
            IF(CURVT.NE.0)GOTO 50
            IF(I.GE.ASLT)FRQ=0.0
            IF(I.LT.ASLT)FRQ=1.0
            GOTO 60
      C
      C CALCULATES THE YEARLY SURVIVAL RATES OF THE NEW VINTAGE.
      C
      50    MT=IFIX(100.*((FLOAT(I)-0.5)/FLOAT(ASLT))+0.5)
            IF(MT.EQ.0)GOTO 350
            M11=MT/10
            M2=MT-10*M11
            IF(M2.EQ.0)M2=10
            IF(M2.EQ.10)M11=M11-1
            M1=45*(CURVT-1)+M11+1
            AVPS=FLOAT(IFIX(GA(L)*PSURV(M1,M2)+.5))
            GOTO 360
      350   AVPS=GA(L)
      360   APS(K+1)=APS(K+1)+AVPS
            ERE(K)=ERE(K)+FIRST-AVPS
            FIRST=AVPS
            K=K+1
            IF((I.NE.J).AND.(L.NE.1))GOTO 59
            REI=1.-PSURV(M1,M2)
      59    IF(K.GT.JX)GOTO 80
            GOTO 70
      60    AVPS=GA(L)*FRQ
            APS(K+1)=APS(K+1)+AVPS
            ERE(K)=ERE(K)+FIRST-AVPS
            FIRST=AVPS
            K=K+1
            IF(K.GT.JX)GOTO 80
      70    CONTINUE
      80    IF(L.GE.JX)GOTO 90
            GOTO 40
      40    CONTINUE
      90    PASUM=0.0
            PASUMC=0.0
            PESUM1=0.0
            PESUM2=0.0
            PSUM1=0.0
            PSUM2=0.0
```

```
      P1=0.0
      A1=0.0
      M=1
      IF(P.NE.1)GOTO 300
      WRITE(6,290)N
290   FORMAT('N2=',I3)
300   DO 100 I=N,120
C
C PERFORMS THE VARIOUS SUMMATIONS REQUIRED TO CALCULATE THE INDICES
C
      PASUM=PASUM+GAA(M)
      PASUMC=PASUMC+GAACF(M)
      PESUM1=(GA(I)-GAA(M))**2+PESUM1
      PESUM2=ABS(GA(I)-GAA(M))+PESUM2
      PSUM1=(GA(I)-GAACF(M))**2+PSUM1
      PSUM2=ABS(GA(I)-GAACF(M))+PSUM2
      A1=GAA(M)**2+A1
      P1=GA(I)**2+P1
      M=M+1
      IF(M.GT.YA)GOTO 270
100   CONTINUE
270   IF(PESUM1.EQ.0.)GOTO 240
      IF(PESUM1.GT..0000001)GOTO 110
      CIC(1)=1./1000000.
      GOTO 260
240   CIC(1)=1./10000000.
      GOTO 260
C
C CALCUATES THE INDICES
C
110   CIC(1)=1./((PASUM/YA)/(SQRT(PESUM1/YA)))
260   CIC(2)=SQRT(PESUM1)/SQRT(A1)
      CIC(3)=(PESUM2/YA)/(PASUM/YA)
      CIC(4)=(PSUM2/YA)/(PASUMC/YA)
      CIC(5)=SQRT(PESUM1/YA)/(SQRT(A1/YA)+SQRT(P1/YA))
      X=0
      XX=0
C
C IF THE COMBINATION OF THE MORTALITY CHARACTERISTICS IS THE
C SAME AS THE ACTUAL COMBINATION, THE VALUES OF THE INDICES
C AND THE OTHER PARAMETERS ARE RECORDED
C
      IF(PESUM2.EQ.0.)CIC(3)=.000001
      IF(PSUM2.EQ.0.)CIC(4)=.000001
      IF((ACTCUR.EQ.CURVT).AND.(ACTASL.EQ.ASLT))X=1
      IF((X.EQ.1).AND.(ACTGRP.EQ.IFLAG(3)))X=2
      IF((X.EQ.2).AND.(YS.EQ.ACTTBL))X=3
      IF(X.NE.3)GOTO 190
      IF((X.EQ.3).AND.(SLOPE.EQ.ACTSLP))XX=1
      IF(XX.NE.1)GOTO 190
      X=0
      ACCI1=1./CIC(1)
      ACCI2=CIC(2)
      ACMCI1=CIC(3)
      ACMCI2=CIC(4)
      ACUI=CIC(5)
      ACASL=ASLT
      ACSLP=SLOPE
      ACTES=CURVT
      ACTOB=N-1
```

```
      ACREI=REI
      XX=0
  190 P=P+1
C
C SELECTS THE NEWLY SELECTED COMBINATION OF THE MORTALITY
C CHARACTERISTICS IF THE VALUES OF THE INDICES PRODUCED BY
C THE NEW COMBINATION ARE BETTER THAN THE ONES ALREADY SELECTED
C DURING PREVIOUS TRIALS
C
      DO 200 L=1,5
      R1=SEL(1,L,L)
      R2=SEL(2,L,L)
      R3=SEL(3,L,L)
      R4=SEL(4,L,L)
      R5=SEL(5,L,L)
      R6=SEL(6,L,L)
      R7=SEL(7,L,L)
      R8=SEL(8,L,L)
      R9=SEL(9,L,L)
      R10=SEL(10,L,L)
      S=AMAX1(R1,R2,R3,R4,R5,R6,R7,R8,R9,R10)
      DO 210 I=1,10
      IF(SEL(I,L,L).NE.S)GOTO 210
      IF(CIC(L).GE.SEL(I,L,L))GOTO 210
      CURS(I,L)=CURVT
      ASLS(I,L)=ASLT
      OBS(I,L)=N-1
      SLOPS(I,L)=SLOPE
      REIS(I,L)=REI
      DO 220 K=1,5
      SEL(I,K,L)=CIC(K)
  220 CONTINUE
      GOTO 200
  210 CONTINUE
  200 CONTINUE
      SLOPE=SLOPE+SLPIN
      MLM=1
      RETURN
      END
C
C
C
C
      SUBROUTINE PEGPR1
C
C THIS SUBROUTINE EXTENDS THE PLANT BALANCES INTO THE TRANSPARENT
C BAND USING AN EXPONENTIAL GROWTH PROFILE
C
      REAL GAA(20),PISA(20),SLOPE,YNTCPT,GAACF(20),PISACF(20),
     +GVALUE(100),GAG(100),PISG(100),SLOPEI,SLOPF,SLPIN
      INTEGER IFLAG(20),YS,Y,YA
      COMMON/FLAG/IFLAG
      COMMON/A/GAA,PISA
      COMMON/SL/SLOPE,YNTCPT,GAACF,PISACF
      COMMON/B/YA
      COMMON/G/YS
      COMMON/Y/Y,P
      COMMON/II/II
      COMMON/GEN/GAG,PISG,YS1
      COMMON/SLP/SLOPEI,SLOPF,SLPIN
```

```
          COMMON/IP/IP
          IF(Y.EQ.1)GOTO 20
          WRITE(7,100)
          WRITE(9,100)
    100   FORMAT(//,'GROWTH PROFILE USED FOR TESTING IS 3')
          WRITE(6,10)
    10    FORMAT(///,'ENTER INITIAL GROWTH RATE (EXPONENTIAL), FINAL',
         +'GROWTH RATE AND THE INCREMENTAL VALUE')
          CALL FREAD(5,'3R:',SLOPEI,SLOPF,SLPIN)
          SLOPE=SLOPEI
    20    IF(IP.EQ.0)SLOPE=SLOPEI
          IF(IFLAG(2).NE.1)GOTO 30
          IF(IFLAG(4).EQ.1)YNTCPT=GAACF(1)
          IF(IFLAG(4).NE.1)YNTCPT=GAA(1)
          GOTO 40
    30    IF(IFLAG(4).EQ.1)YNTCPT=PISACF(1)
          IF(IFLAG(4).NE.1)YNTCPT=PISA(1)
    40    II=0
          GVALUE(1)=YNTCPT/SLOPE
          DO 50 I=2,YS
          GVALUE(I)=GVALUE(I-1)/SLOPE
    50    CONTINUE
          K=YS
          IF(IFLAG(2).NE.1)GOTO 60
          DO 70 I=1,YS
          GAG(K)=FLOAT(IFIX(GVALUE(I)+.5))
          PISG(K)=0.0
          K=K-1
    70    CONTINUE
          GOTO 80
    60    DO 90 I=1,YS
          PISG(K)=FLOAT(IFIX(GVALUE(I)+.5))
          GAG(K)=0.0
          K=K-1
    90    CONTINUE
    80    Y=1
          IP=1
          RETURN
          END
nd of file
```

APPENDIX III

Variables Used in the Monte Carlo Simulator

1. ASL - The average service life to be used for the simulation
2. CFRQ - Iowa type cumulative retirement frequency distribution curve.
3. CFRQD - Retirement frequency distribution curve.
4. DSEED - Seed number used for the random number generation.
5. ERE - Expected retirements in the current year from all the vintages to date.
6. FRQCUM - Cumulative frequency distribution table of the Iowa type curve.
7. GA - Plant additions.
8. NCURV - The type number of the curve to be used for the simulation.
9. NYRS - Number of years for which the account is to be simulated.
10. PIS - Plant in service (plant balance).
11. PSURV - Standard Iowa type survival tables.
12. RET - Vintage retirements.
13. SLOPE - Growth rate of the plant account.
14. START - Starting value of the plant balance, ie., the plant balance at the end of the first year of the account. If the growth profile used is stationary, this is the value at which the plant balance is to be held stationary.

APPENDIX IV

Computer Program of the Monte Carlo Simulator

```
      REAL START,FRQCUM(1395,10),PSURV(1395,10)
C
C THIS IS THE MAIN PROGRAM OF THE MONTE CARLO SIMULATOR. THIS
C SECTION OF THE PROGRAM ASSIGNS THE INPUT AND THE OUTPUT DEVICES,
C READS THE INPUT PARAMETERS AND CALLS THE APPROPRIATE SUBROUTINE
C TO SIMULATE THE PLANT ACCOUNT EITHER DETERMINISTICALLY OR
C DETERMINISTICALLY
C
      INTEGER NYRS,NCURV,IFLAG(10),ASL
      DOUBLE PRECISION DSEED,X
      COMMON/FRQ/FRQCUM
      COMMON/FLAG/IFLAG
      COMMON/IN/START,NYRS,NCURV,ASL
      COMMON/P/PSURV
      COMMON/DS/DSEED,X
C
C ASSIGNS THE INPUT AND THE OUTPUT DEVICES
C
      CALL FTNCMD('ASSIGN 2=KRPR:ICURVE',20)
      CALL FTNCMD('ASSIGN 3=KRPR:FRQCUM',20)
      CALL FTNCMD('ASSIGN 7=KRPR:MONTEOUT(*L+1)',28)
      CALL FTNCMD('ASSIGN 8=KRPR:MONTEOUTU(*L+1)',29)
C
C READS THE INPUT DATA
C
      DO 10 I=1,1395
      READ(3,20)(FRQCUM(I,J),J=1,10)
      READ(2,20)(PSURV(I,J),J=1,10)
20    FORMAT(10F11.6)
10    CONTINUE
      X=624.
      CALL FREAD(5,'I:',MNUM)
      MNUM=MNUM-1
110   CONTINUE
      IFLAG(3)=0
      MNUM=MNUM+1
      CALL FREAD(5,'I:',IFLAG(1))
      CALL FREAD(5,'1R:',START)
      CALL FREAD(5,'3I:',NYRS,NCURV,ASL)
      J=IFLAG(1)
      WRITE(7,60)
      WRITE(8,60)
60    FORMAT('1','*******************************************')
      WRITE(8,130)MNUM
      WRITE(7,130)MNUM
130   FORMAT(///,3X,'MODEL NUMBER=',I4)
C
C CALLS THE APPROPRIATE SUBROUTINE TO SIMULATE THE ACCOUNT
C EITHER DETERMINISTICALLY OR STOCHASTICALLY
C
      GOTO(30,40),J
30    CALL DETRMN
      GOTO 50
 40   CALL PRBLST
 50   CONTINUE
      STOP
      END
C
C
C
C
```

```
      SUBROUTINE DETRMN
      INTEGER IFLAG(10)
      COMMON/FLAG/IFLAG
C
C THIS SUBROUTINE SIMULATES THE PLANT ACCOUNTS DETERMINISTICALLY
C
      CALL FREAD(5,'I:',IFLAG(2))
      J=IFLAG(2)
      WRITE(8,50)
      WRITE(7,50)
50    FORMAT(////,'DETERMINISTIC DATA GENERATION')
C
C CALLS THE SUBROUTINE TO CALCULATE THE PLANT BALANCES AS PER
C THE SPECIFIED GROWTH PROFILE
C
      GOTO(10,10,20),J
10    CALL SLGRP1
      GOTO 40
20    CALL PEGP11
      GOTO 40
40    CONTINUE
C
C CALLS THE SUBROUTINE TO SIMULATE THE PLANT ADDITIONS AND
C RETIREMENTS TO MMAINTAIN THE REQUIRED PLANT BALANCES
C
      CALL SIMLT1
      RETURN
      END
C
C
C
C
      SUBROUTINE SLGRP1
C
C THIS SUBROUTINE CALCULATES THE PLANT BALANCES TO CONFORM EITHER
C TO A STATIONARY PROFILE OR A LINEAR GROWTH PROFILE (WITH
C POSITIVE SLOPE).
C
      REAL START,SLOPE,PIS(60)
      INTEGER IFLAG(10),NYRS,NCURV,ASL
      COMMON/FLAG/IFLAG
      COMMON/IN/START,NYRS,NCURV,ASL
      COMMON/PIS/PIS
      IF(IFLAG(2).EQ.1)GOTO 10
      CALL FREAD(5,'1R:',SLOPE)
      GOTO 30
10    SLOPE=0.0
30    CONTINUE
      M=0
      DO 40 I=1,NYRS
      PIS(I)=FLOAT(IFIX((START+SLOPE*M)+.5))
      M=M+1
40    CONTINUE
      WRITE(8,50)
      WRITE(7,50)
50    FORMAT(//,'THE INPUT VARIALBLES OF THE CURVE ARE:')
      WRITE(8,60)START,NYRS,NCURV,ASL,SLOPE
      WRITE(7,60)START,NYRS,NCURV,ASL,SLOPE
60    FORMAT(//,'THE GROWTH PROFILE USED IS LINEAR',//,
     +'START VALUE=',F9.0,5X,'TOTAL # YRS=',I4,5X,'CURVE NUMBER='
```

```
     +,I3,5X,'ASL=',I3,5X,'SLOPE=',F9.3)
      RETURN
      END
C
C
C
C
      SUBROUTINE PEGP11
C
C THIS SUBROUTINE GENERATES THE PLANT BALANCES TO CONFORM TO AN
C EXPONENTIAL GROWTH PROFILE
C
      REAL START,GRRT,PIS(60)
      INTEGER IFLAG(10),NYRS,NCURV,ASL
      COMMON/FLAG/IFLAG
      COMMON/IN/START,NYRS,NCURV,ASL
      COMMON/PIS/PIS
      CALL FREAD(5,'1R:',GRRT)
      PIS(1)=START
      DO 20 I=2,NYRS
      PIS(I)=FLOAT(IFIX((PIS(I-1)*GRRT)+.5))
20    CONTINUE
      WRITE(8,30)
      WRITE(7,30)
30    FORMAT(//,'THE INPUT VARIABLES OF THE CURVE ARE')
      WRITE(8,40)START,NYRS,NCURV,ASL,GRRT
      WRITE(7,40)START,NYRS,NCURV,ASL,GRRT
40    FORMAT(//,'THE GROWTH PROFILE USED IS POSITIVE EXPONENTIAL',
     +' - TYPE 1',//,'START VALUE=',F9.0,5X,'TOTAL # OF YRS=',I4,5X,
     +'CURVE NUMBER=',I3,5X,'ASL=',I3,5X,'GROWTH RATE=',F7.3)
      RETURN
      END
C
C
C
C
      SUBROUTINE SIMLT1
C
C THIS SUBROUTINE SIMULATES THE PLANT ADDITIONS AND RETIREMENTS
C TO SATISFY THE REQUIRED GROWTH PROFILE AND THE GROWTH RATE
C
      REAL START,PSURV(1395,10),GAA(60),PIS(60),SUM(60)
     +,GA(60),RET(60),RETR(60),ERE(100)
      INTEGER IFLAG(10),NYRS,NCURV,ASL,ASLT,ROW,COLMN,CURVT
      COMMON/FLAG/IFLAG
      COMMON/P/PSURV
      COMMON/IN/START,NYRS,NCURV,ASL
      COMMON/PIS/PIS
      DO 10 I=1,60
      SUM(I)=0.0
      RET(I)=0.0
10    CONTINUE
      DO 200 I=1,100
      ERE(I)=0.
200   CONTINUE
      CURVT=NCURV
      ASLT=ASL
C
C CALCULATES THE PERCENT SURVIVING AT THE END OF THE FIRST
C YEAR OF INSTALLATION
```

```
C
      KHALF=IFIX(100.*(.5/FLOAT(ASLT))+.5)
      NN=KHALF-(KHALF/10)*10
      IF(NN)20,30,20
30    ROW=IFIX(KHALF/10.)+(CURVT-1)*45
      COLMN=10
      GOTO 40
20    ROW=(CURVT-1)*45+(KHALF/10)+1
      COLMN=KHALF-(KHALF/10)*10
40    IF(KHALF.EQ.0)PSUR=1.
      IF(KHALF.NE.0)PSUR=PSURV(ROW,COLMN)
      DO 50 I=1,60
C
C CALCULATES THE PLANT ADDITIONS
C
      GA(I)=FLOAT(IFIX(((PIS(I)-SUM(I)+ERE(I))/PSUR)+.5))
      K=I
      FIRST=GA(I)
      DO 60 J=1,60
      MT=IFIX(100.*((FLOAT(J)-.5)/FLOAT(ASLT))+.5)
      M11=MT/10
      M2=MT-10*M11
      IF(M2.EQ.0)M2=10
      IF(M2.EQ.10)M11=M11-1
      M1=45*(CURVT-1)+M11+1
      AVPS=FLOAT(IFIX((GA(I)*PSURV(M1,M2))+.5))
      SUM(K+1)=SUM(K+1)+AVPS
C
C CALCULATES THE VINTAGE RETIREMENTS
C
      RET(J)=FIRST-AVPS
      ERE(K)=ERE(K)+RET(J)
      FIRST=AVPS
      K=K+1
      N=J
      IF(AVPS.EQ.0.)GOTO 70
60    CONTINUE
C
C PRINTS THE ADDITIONS, RETIREMENTS AND THE PLANT BALANCES
C FOR THAT YEAR
C
70    WRITE(7,80)I,GA(I),ERE(I),PIS(I)
80    FORMAT(////,'YEAR',I3,10X,'GROSS ADDITIONS=',F9.0,10X,
     +'RETIREMENTS=',F9.0,10X,'PLANT BALANCE=',F9.0)
      RETR(I)=ERE(I)
      L=1
      L1=L+1
      L2=L+2
C
C PRINTS THE VINTAGE RETIREMENTS
C
      WRITE(7,160)
160   FORMAT(//,'VINTAGE RETIREMENTS:',/)
      DO 100 M=1,20
      WRITE(7,90)L,RET(L),L1,RET(L1),L2,RET(L2)
      IF(L2.GE.N)GOTO 110
      L=L+3
      L1=L+1
      L2=L+2
100   CONTINUE
```

```
90    FORMAT(3X,3('(',I2,')',F9.0,15X))
110   CONTINUE
      IF(I.GE.NYRS)GOTO 120
50    CONTINUE
 120  WRITE(8,150)
      WRITE(7,151)
C
C PRINTS THE SUMMARY OF THE SIMULATED ACCOUNT
C
      DO 140 I=1,NYRS
      WRITE(8,130)I,GA(I),RETR(I),PIS(I)
      WRITE(7,130)I,GA(I),RETR(I),PIS(I)
130   FORMAT(2X,I3,10X,F9.0,8X,F14.0,9X,F9.0)
140   CONTINUE
150   FORMAT(////,'YEAR',5X,'PLANT ADDITIONS',5X,'PLANT RETIREMENTS'
     +,5X,'PLANT BALANCES')
151   FORMAT('1',/,'YEAR',5X,'PLANT ADDITIONS',5X,'PLANT RETIREMENTS'
     +,5X,'PLANT BALANCES')
      RETURN
      END
C
C
C
C
      SUBROUTINE PRBLST
C
C THIS SUBROUTINE SIMULATES THE PLANT ACCOUNT STOCHASTICALLY
C
      INTEGER IFLAG(10)
      COMMON/FLAG/IFLAG
      CALL FREAD(5,'I:',IFLAG(2))
      J=IFLAG(2)
      WRITE(8,50)
      WRITE(7,50)
50    FORMAT(////,'STOCHASTIC DATA GENERATION')
C
C CALLS THE APPROPRIATE SUBROUTINE TO CALCULATE THE PLANT
C BALANCES AS PER THE SPECIFIED GROWTH PROFILE
C
      GOTO(10,10,20),J
10    CALL SLGRP1
      GOTO 40
20    CALL PEGP11
      GOTO 40
40    CONTINUE
C
C CALLS THE SUBROUTINE TO STOCHASTICALLY SIMULATE THE RETIREMENTS,
C PLANT ADDITIONS AND THE PLANT BALANCES
C
      CALL SIMLT2
      RETURN
      END
C
C
C
C
      SUBROUTINE SIMLT2
C
C THIS SUBROUTINE STOCHASTICALLY SIMULATES THE PLANT ADDITIONS,
C PLANT RETIREMENTS AND THE PLANT BALANCES.
```

```
C
      REAL PIS(60),CFRQ(100),R(170000),GA(60),SUMM(100),RE(100)
     +,RET(100),FRQCUM(1395,10),ERE(100),CFRQD(100),C(101)
      INTEGER IFLAG(10),NYRS,NCURV,ASL,ASLT,CURVT
      DOUBLE PRECISION DSEED,X
      COMMON/FLAG/IFLAG
      COMMON/DS/DSEED,X
      COMMON/IN/START,NYRS,NCURV,ASL
      COMMON/PIS/PIS
      COMMON/FRQ/FRQCUM
      CALL FREAD(5,'R:',DSEED)
      IF(DSEED.LE.1.)DSEED=X
      WRITE(7,15)DSEED
      WRITE(8,15)DSEED
15    FORMAT(//,'SEED NUMBER USED=',F16.4)
      ASLT=ASL
      N=1
      CURVT=NCURV
      DO 300 I=1,101
      C(I)=0.
300   CONTINUE
      DO 10 I=1,100
C
C CALCULATES THE RETIREMENT FREQUENCY
C
      MT=IFIX(100.*((FLOAT(I)-.5)/FLOAT(ASLT))+.5)
      M11=MT/10
      M2=MT-10*M11
      IF(M2.EQ.0)M2=10
      IF(M2.EQ.10)M11=M11-1
      IF(M11.GT.44)GOTO 20
      M1=45*(CURVT-1)+M11+1
      CFRQ(I)=FRQCUM(M1,M2)
      C(I+1)=CFRQ(I)
      CFRQD(I)=CFRQ(I)-C(I)
      N=N+1
10    CONTINUE
20    CONTINUE
      DO 90 I=1,100
      SUMM(I)=0.0
      RE(I)=0.0
      ERE(I)=0.
90    CONTINUE
      PISS=0.0
      DO 30 I=1,60
      MM=I
      SUM=0.
C
C CALCULATES THE PLANT ADDITIONS
C
      GA(I)=FLOAT(IFIX((PIS(I)-PISS+ERE(I))/(1.-CFRQ(1))+.5))
      KK=IFIX(GA(I))
C
C CALLS THE SUBROUTINE TO GENERATE THE RANDOM NUMBERS
C
      CALL RANDOM(KK,R)
      KK=IFIX(GA(I))
      DO 50 L=1,60
      RET(L)=0.
50    CONTINUE
```

```
C
C CALCULATES THE VINTAGE RETIREMENTS
C
      DO 40 J=1,KK
      DO 60 L=1,N
      LL=L
      IF(R(J).LE.CFRQ(L))GOTO 70
60    CONTINUE
70    RET(LL)=RET(LL)+1.
40    CONTINUE
      DO 80 L=1,60
      SUM=SUM+RET(L)
      SUMM(MM)=SUMM(MM)+GA(I)-SUM
      RE(MM)=RE(MM)+RET(L)
      IF(MM.GE.NYRS)GOTO 210
      MM=MM+1
80    CONTINUE
210   CONTINUE
      II=I
      DO 220 L=1,N
      ERE(II)=FLOAT(IFIX((GA(I)*CFRQD(L))+.5))+ERE(II)
      II=II+1
220   CONTINUE
      PISS=SUMM(I)
C
C PRINTS THE PLANT ACCOUNT TRANSACTIONS OF THAT YEAR
C
      WRITE(7,100)I,GA(I),RE(I),SUMM(I)
100   FORMAT(////,'YEAR',I3,10X,'GROSS ADDITIONS=',F9.0,10X,
     +'RETIREMENTS=',F9.0,10X,'PLANT BALANCE=',F9.0)
      L=1
      L1=L+1
      L2=L+2
      S=0.
C
C PRINTS THE VINTAGE RETIREMENTS
C
      WRITE(7,110)
110   FORMAT(//,'VINTAGE RETIREMENTS:',/)
      DO 120 J=1,20
      S=S+RET(L)+RET(L1)+RET(L2)
      WRITE(7,130)L,RET(L),L1,RET(L1),L2,RET(L2)
130   FORMAT(3X,3('(',I2,')',F9.0,15X))
      IF(S.GE.GA(I))GOTO 140
      L=L+3
      L1=L+1
      L2=L+2
120   CONTINUE
140   CONTINUE
      IF(I.GE.NYRS)GOTO 150
30    CONTINUE
150   CONTINUE
      WRITE(8,160)
      WRITE(7,161)
160   FORMAT(////,'YEAR',5X,'PLANT ADDITIONS',5X,'PLANT RETIREMENTS',
     +5X,'PLANT BALANCES')
161   FORMAT('1',/,'YEAR',5X,'PLANT ADDITIONS',5X,'PLANT RETIREMENTS',
     +5X,'PLANT BALANCES')
      DO 170 I=1,60
      WRITE(8,180)I,GA(I),RE(I),SUMM(I)
```

```
      WRITE(7,180)I,GA(I),RE(I),SUMM(I)
  180 FORMAT(1X,I3,9X,F9.0,13X,F9.0,10X,F9.0)
      IF(I.GE.NYRS)GOTO 200
  170 CONTINUE
  200 CONTINUE
      RETURN
      END
C
C
C
C
      SUBROUTINE RANDOM(KK,R)
C
C THIS SUBROUTINE GENERATES THE RANDOM NUMBERS REQUIRED FOR THE
C SIMULATION
C
      REAL R(KK)
      DOUBLE PRECISION DSEED,X
      COMMON/DS/DSEED,X
C
C GGUBS IS A SUBROUTINE AVAILABLE ON THE SYSTEM LIBRARY
C (*IMSLLIB). IT GENERATES UNIFORMLY DISTRIBUTED RANDOM NUMBERS
C IN THE RANGE OF 0 TO 1
C
      CALL GGUBS(DSEED,KK,R)
      X=DSEED
      RETURN
      END
End of file
```

APPENDIX V

Numbering of Iowa Curves

Iowa Curve Type	Number Used
L0	01
L0.5	02
L1	03
L1.5	04
L2	05
L3	06
L4	07
L5	08
S(-0.5)	09
S0	10
S0.5	11
S1	12
S1.5	13
S2	14
S3	15
S4	16
S5	17
S6	18
R0.5	19
R1	20
R1.5	21
R2	22
R2.5	23

R3	24
R4	25
R5	26
O1	27
O2	28
O3	29
O4	30
O5	31

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